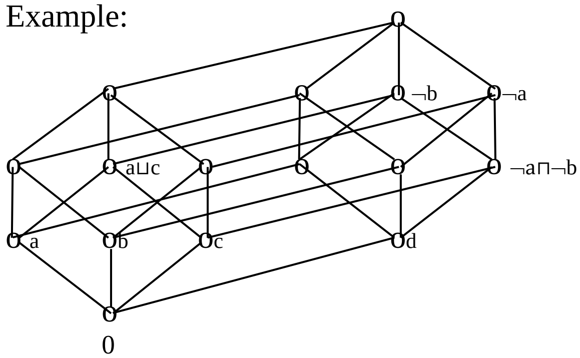
4. Applications of Boolean algebra

Based on Landman 2020: Iceberg Semantics of Count Nouns and Mass Nouns

4.1. Boolean background

I assume semantic interpretation domains which are complete Boolean algebras: structures **B** with domain B, ordered by part-of relation ⊑, with minimum 0, and operations ⊔ of join, supremum or sum and ⊓ of meet, infimum or overlap and ¬ of complement or remainder.

Example:



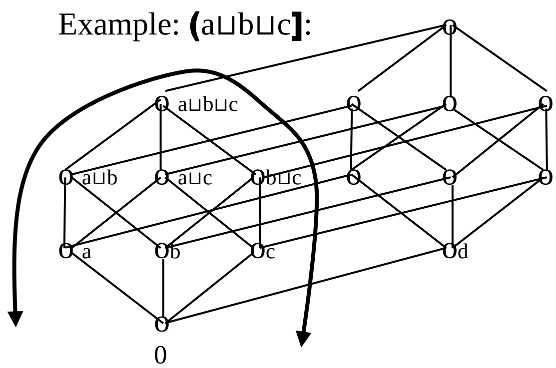
It is useful to distinguish between elements of B and *objects* in B, where objects are non-null elements:

Let **B** be a Boolean algebra and X ⊆ B.
▻ X^+, the set of *objects* in X, is given by: $X^+ = X - \{0\}$

Following Grätzer 1978, I use half-closed interval notation for Boolean part sets:

Boolean part set: Let x ∈ B, X ⊆ B
▻ **(**x**]** = {b ∈ B: b ⊑ x} The set of all Boolean parts of x
▻ **(**X**]** = **(**⊔X**]** The set of all Boolean parts of ⊔X

Example: **(**a⊔b⊔c**]**:

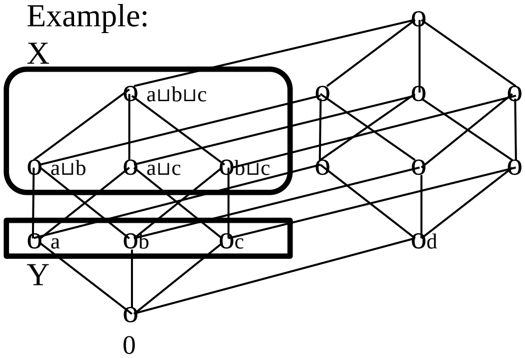


Closure and generation under sum ⊔: Let X,Y ⊆ B.
▻ *X = {b ∈ B: for some Y ⊆ X: b = ⊔Y} The set of all sums of elements of X

▻ Y *generates* X *under* ⊔ iff $X \subseteq {}^*Y$ and $\sqcup Y = \sqcup X$

All elements of X are sums of elements of Y, and X and Y have the same supremum.

Example:

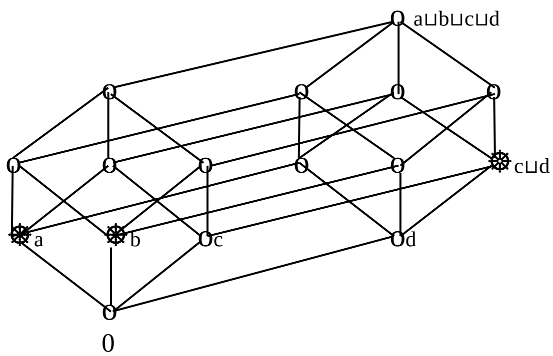


$^*Y = Y \cup X$, $\sqcup Y = \sqcup X$
hence Y generates X under ⊔

Let $X \subseteq B$ and $b \in B$

▻ X is a *partition* of b iff X is a non-empty disjoint subset of $(b]^+$ such that $\sqcup X = b$

An example is given in figure 1.8:



{a, b, c⊔d} is a partition of a⊔b⊔c⊔d

Atomicity:

Let $a \in B$, $X \subseteq B$.

a is an X-*atom* iff $a \in X^+$ and for every $x \in X^+$: if $x \sqsubseteq a$ then $x = a$.
$ATOM_X$ is the set of X-atoms.

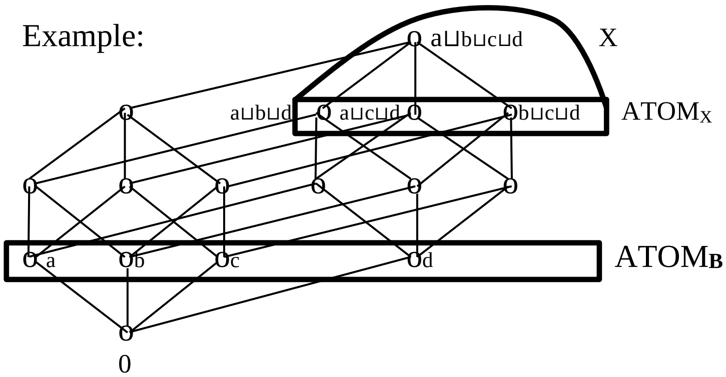
The set of X-atoms is the set of objects of X that are minimal objects in X, i.e. that have no proper parts that are also in X. If 0 is not in X, then $ATOM_X$ is just the set of minimal elements in X.

The notions I define here are the *generalizations* of the standard Boolean notions to arbitrary subsets of B. This means that they are notions that relate to the bottom of *subsets* of B, not just to the bottom of B. The difference is all important in this book.

The standard Boolean notions of atoms you get by taking X to be B.

a is an *atom in* **B** iff a is a B-atom.
$ATOM_{\mathbf{B}} = ATOM_B$

Example:



X is *atomic* iff for every $x \in X^+$ there is an $a \in ATOM_X$: $a \sqsubseteq x$
X is *atomistic* iff for every $x \in X$ there is a set $A \subseteq ATOM_X$: $x = \sqcup A$
X is *atomless* iff $ATOM_X = Ø$

X is *atomic* means that every element of X^+ has at least one part that is an X-atom.
X is *atomistic* iff every element of X is the sum of X-atoms.
X is *atomless* iff there are no X-atoms, i.e. X has no minimal (mereological) parts

Again the standard Boolean notions are the case where X is B:
B is *atomic/atomistic/atomless* iff B is *atomic/atomistic/atomless*

We proved in chapter 4:
Theorem: if **B** is a complete Boolean algebra then **B** is atomic iff **B** is atomistic.

Fact: This does not generalize to arbitrary subsets: if **B** is a complete Boolean algebra and $X \subseteq B$, X can be atomic without being atomistic.

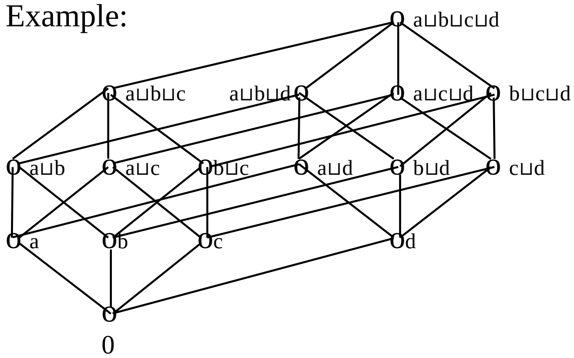
Disjointness and overlap: Let $x,y \in B$, $X,Y \subseteq B$

We define overlap for objects

x and y *overlap*: **overlap**(x,y) iff $x \sqcap y \in B^+$
x and y are *disjoint*: **disjoint**(x,y) iff ¬**overlap**(x,y)
X *overlaps*: **overlap**(X) iff for some $x, y \in X$: **overlap**(x,y)
X is *disjoint*: **disjoint**(X) iff ¬**overlap**(X)
X and Y *overlap*: **overlap**(X,Y) iff for some $x \in X$, $y \in Y$: **overlap**(x,y)
X and Y are *disjoint*: **disjoint**(X,Y) iff ¬**overlap**(X,Y)

Two elements overlap if they share a part which is an object, i.e. non-null part. This implies that they themselves are objects. Two elements are disjoint if they do not overlap.
A set overlaps if two of its elements overlap. A set is disjoint if none of its elements overlap.
Two sets overlap if some element of the one overlaps some element of the other. Two sets are disjoint if no element of the one overlaps any element of the other.

Example:



{a⊔b, c⊔d} is disjoint

{a⊔b, b⊔d} overlaps

{a,b,c,d} is disjoint

A theorem that motivates the approach to Iceberg semantics developed in this book is:

Theorem: if X is a *disjoint subset* of B, then *X forms a complete atomic Boolean algebra.

In complete atomic Boolean algebras counting is counting of atoms:

Let **B** be a complete atomic Boolean algebra and b ∈ B.
▻ The *cardinality of* b, |b|, is given by: $|b| = |ATOM_b|$

The cardinality of b is the cardinality of the set of b's atomic parts.
An example is given in figure 1.11:

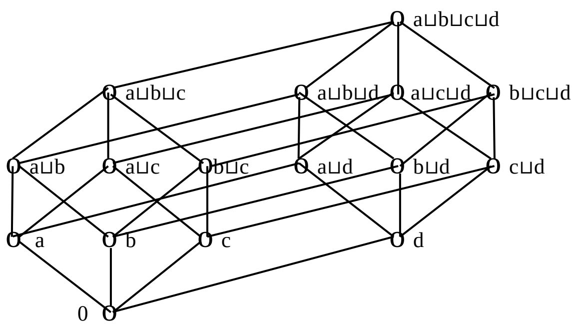


Figure 1.11

$ATOM_{a⊔b⊔c⊔d}$ = {a,b,c,d}	|a⊔b⊔c⊔d| = 4
$ATOM_{a⊔b⊔c}$ = {a,b,c}	|a⊔b⊔c| = 3
$ATOM_{a⊔b}$ = {a,b}	|a⊔b| = 2
$ATOM_a$ = {a}	|a| = 1
$ATOM_0$ = Ø	|0| = 0

▻ If **B** is a complete atomic Boolean algebra, the *cardinality function* is the function that maps every object in B onto its cardinality: **card** = λz.|z|

4.2. Boolean semantics for count nouns (Mountain semantics, following Link 1983, 1984)

▹ A *count domain* is a complete atomic Boolean algebra **B**.

Singular nouns are interpreted as sets of atoms:

cat → CAT_w with $\text{CAT}_w \subseteq \text{ATOM}_{\mathbf{B}}$

Plural nouns are interpreted as the closure under sum of singular noun denotations:

cats → $*\text{CAT}_w$

Let ATOM_B = {r, e, s, f} and let *Ronya* → r *Emma* → e *Shunra* → s *Fido* → f
Figure 2.1. shows CAT_w and $*\text{CAT}_w$:

cat → $\text{CAT}_w = \{r, e, s\} \subseteq \text{ATOM}_B$ *cats* → $*\text{CAT}_w = \{0, r, e, s, r⊔e, r⊔s, e⊔s, r⊔e⊔s\}$

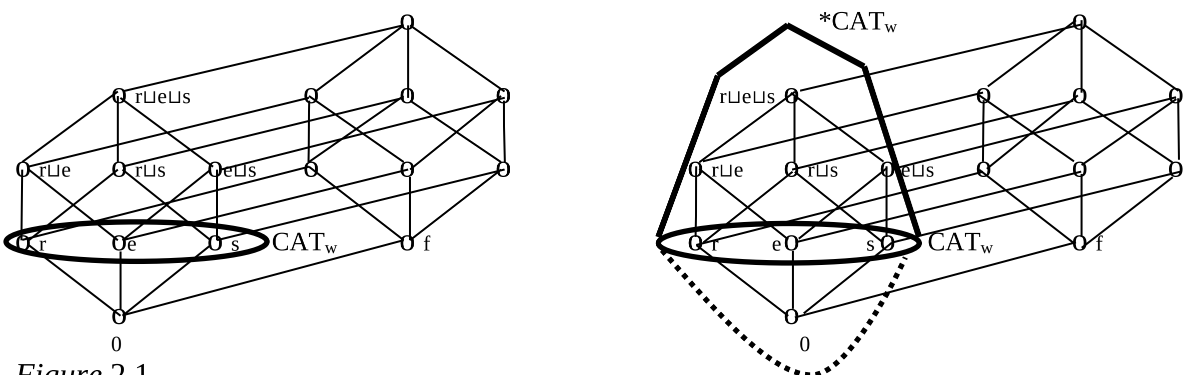


Figure 2.1

Mountain semantics is so-called because, as the picture illustrates, plural denotations are *mountains* rising up from the atomic seabed.

Link 1983: interpret grammatical plurality via an operation of semantic pluralization (linking semantic plurality to cumulativity and distributitivy, see Landman 1991). It makes plural denotations into *mountains* rising up from the atomic seabed.

We assume that DP conjunction allows, besides the standard generalized conjunction interpretation based on $\wedge$, an interpretation as *sum conjunction*:

▹ *Sum conjunction*: *and* → $\lambda y \lambda x.\ x \sqcup y$ (at type e)

With this, we allow:

ronya and emma and shunra → $r \sqcup e \sqcup s$ $(\ \lambda y \lambda x.x \sqcup y(r,(\lambda y \lambda x.x \sqcup y(e,s)))\ )$
are cats → $*\text{CAT}_w$

Hence, (1a) gets interpreted as (1b):

(1) a. Ronya and Emma and Shunra are cats.
 b. $*CAT_w(r \sqcup e \sqcup s)$
 c. $CAT_w(r) \wedge CAT_w(e) \wedge CAT_w(s)$

Lemma: Given that $CAT_w \subseteq ATOM_B$, (1b) is equivalent to (1c).

Proof: 1. If r, e, s $\in CAT_w$ then {r, e, s} $\subseteq$ CAT, hence $r \sqcup e \sqcup s \in *CAT$.
 2. Assume $r \sqcup e \sqcup s \in *CAT_w$. Then, by definition of *, for some $X \subseteq CAT_w$: $r \sqcup e \sqcup s = \sqcup X$. In a complete atomic Boolean algebra there is only one set of atoms X such that $r \sqcup e \sqcup s = \sqcup X$, namely {r, e, s}.
Hence $\{r, e, s\} \subseteq CAT_w$, i.e. r, e, s $\in CAT_w$.

Numerical predicates (the basis of the analysis was an example in chapter 1)

I follow here the compositional analysis of numerical phrases of Landman 2004. The semantics is based on the idea that the semantic composition takes place at the lowest available type.
For number expressions like *one, two, three* this is type n of numbers:

Numbers: type n
three → 3
eight → 8

I assume that expressions *at most/less than/at least/more than/exactly* denote two place relations between numbers, and *between…and…*a three place relation between numbers:

Two place number relations:		type <n,<n,t>>
at most → $\leq$	($= \lambda m \lambda n. n \leq m$)	*less than* → $<$
at least → $\geq$		*more than* → $>$
exactly → $=$		
Three place number relations:		type <n,<n,<n,t>>>
between…and… → $\lambda k \lambda m \lambda n.\ k \leq n \leq m$		

Number relations combine with numbers through *functional application* to form number predicates:

number predicate = (***number relation***(***number***))

number predicates: type <n,t>
at most three → $\lambda n.\ n \leq 3$
less than three → $\lambda n.\ n < 3$
at least three → $\lambda n.\ n \geq 3$
more than three → $\lambda n.\ n > 3$
exactly three → $\lambda n.\ n = 3$
between three and eight → $\lambda n.\ 3 \leq n \leq 8$

I assume that bare number expressions like *three* too have an interpretation at the type of number predicates, either because the number predicate *three* contains a null relational phrase, or because *three* typeshifts to this type. In either case assume that the default interpretation of *three* at type <n,t> involves the number relation =:

three → $\lambda n.n=3$ of type <n,t>

Hence, in the compositional analysis of numerical phrases the type of number predicates <n,t> is the pivotal type: the semantic derivation numerical phrases (and measure phrases) involves a number predicate of type <n,t>. From the number predicate of type <n,t>, we derive a numerical predicate of type <e,t>:

more than three → $\lambda n.n > 3$ of type <n,t>
more than three → $\lambda x.|x| > 3$ of type <e,t>

The derivation becomes visible through the following equivalences.

$\lambda x.|x| > 3$	= [backwards λ-conversion on |x|]
$\lambda x.(\lambda n.n > 3(|x|))$	= [backwards λ-conversion on x]
$\lambda x.(\lambda n.n > 3(\lambda z.|z|(x)))$	= [definition of function composition, $g \circ f = \lambda x.g(f(x))$]
$\lambda n.n > 3 \circ \lambda z.|z|$	= [definition of **card**]
$\lambda n.n > 3 \circ$ **card**	

Thus:

numerical predicate = ***number predicate*** ∘ **card**
<e,t> <n,t> <e,n>

In fact, since **card** is a function of type <e,n>, the same type as measure functions like $\mathbf{liter}_{wt}$, the function that maps at wt objects onto their volume at wt in liters, this semantic composition principle generalizes to measures in general:

measure predicate = ***number predicate*** ∘ **measure**
<e,t> <n,t> <e,n>

Thus, we derive through composition with a lexically realized measure a measure predicate of type <e,t>:

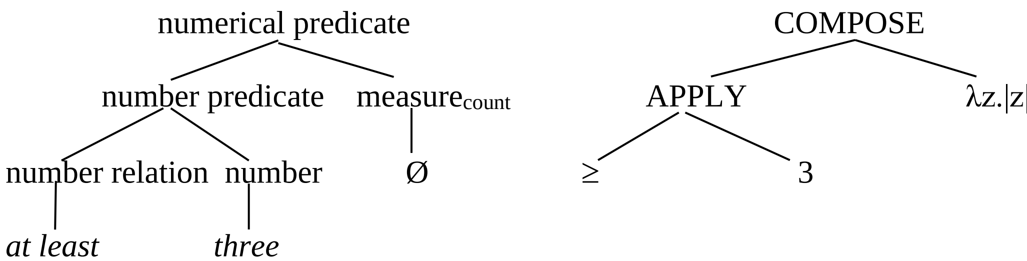
at least three liters → $\lambda n.n \geq 3 \circ \mathbf{liter}_{wt} = \lambda x.\mathbf{liter}_{wt}(x) \geq 3$

and we derive through composition with the cardinality measure **card**, which is not lexically realized, the following numerical predicates:

numerical predicates type <e,t>
at most three → $\lambda x.\ |x| \leq 3$
less than three → $\lambda x.\ |x| < 3$

at least three → $\lambda x.\ |x| \geq 3$
more than three → $\lambda x.\ |x| > 3$
exactly three → $\lambda x.\ |x| = 3$
between three and eight → $\lambda x.\ 3 \leq |x| \leq 8$
three → $\lambda x.\ |x| = 3$

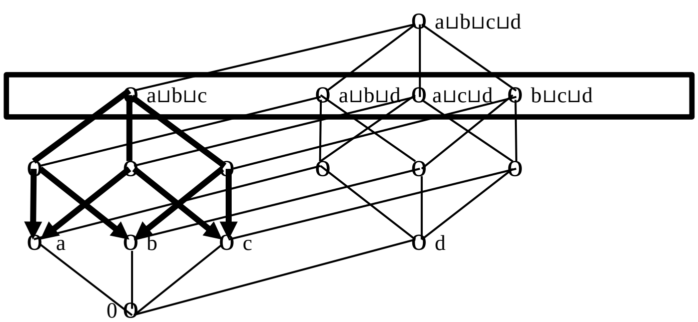
Again, one can assume, as I did in Landman 2004, that the numerical phrase is formed from the number predicate and a null measure which is by default interpreted as the cardinality function:



But it is also possible not to realize a null measure node, and realize the <e,t> interpretation via a type shift:

shift	$n \Rightarrow (=(n))$		$N \Rightarrow$	$N \circ \lambda z.|z|$
type	n	$\langle n,t\rangle$	$\langle n,t\rangle$	$\langle e,t\rangle$

example: *three* → $\lambda x.|x|=3$

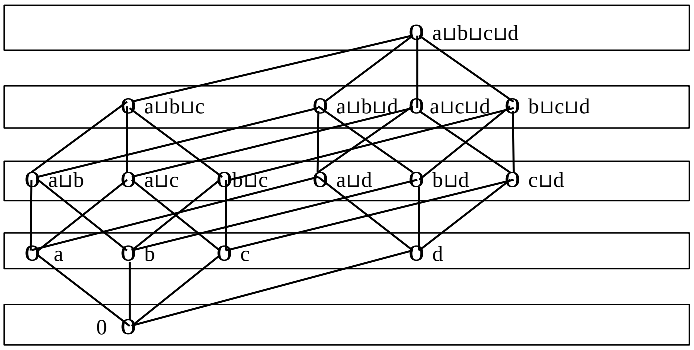


$\lambda x.|x|=3 \quad = \quad \lambda x.|ATOM_x|=3$

$ATOM_{a \sqcup b \sqcup c} = \{a, b, c\}$, $ATOM_{a \sqcup b \sqcup d} = \{a, b, d\}$
$ATOM_{a \sqcup c \sqcup d} = \{a, c, d\}$, $ATOM_{b \sqcup c \sqcup d} = \{b, c, d\}$.

$\lambda x.|x|=3 \quad = \quad \{a \sqcup b \sqcup c,\ a \sqcup b \sqcup d,\ a \sqcup c \sqcup d,\ b \sqcup c \sqcup d\}$

Hence, $\lambda x.|x|=n$ denotes the set of elements of **B** at height n:

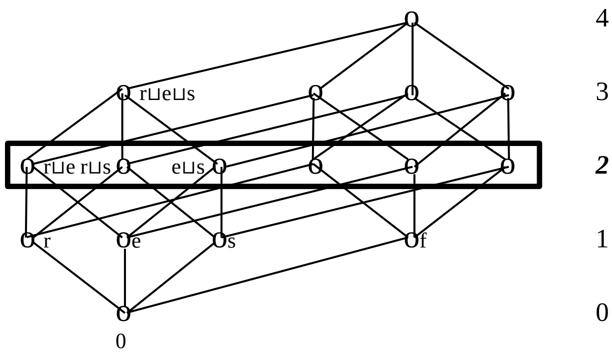


$\lambda x.|x| = 4$	=	$\{a \sqcup b \sqcup c \sqcup d\}$	4
$\lambda x.|x| = 3$	=	$\{a \sqcup b \sqcup c,\ a \sqcup b \sqcup d,\ a \sqcup c \sqcup d,\ b \sqcup c \sqcup d\}$	3
$\lambda x.|x| = 2$	=	$\{a \sqcup b,\ a \sqcup c,\ a \sqcup d,\ b \sqcup c,\ b \sqcup d,\ c \sqcup d\}$	2
$\lambda x.|x| = 1$	=	$\{a, b, c, d\}$	1
$\lambda x.|x| = 0$	=	$\{0\}$	0

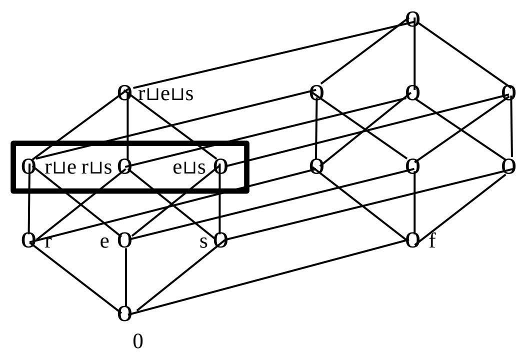
numerical predicate* + *noun phrase* = *numerical predicate* $\cap$ *noun phrase

exactly two cats $\rightarrow \lambda x.|x| = 2 \ \cap\ {}^*CAT_{wt} = \lambda x.{}^*CAT_{wt}(x) \wedge |x| = 2$

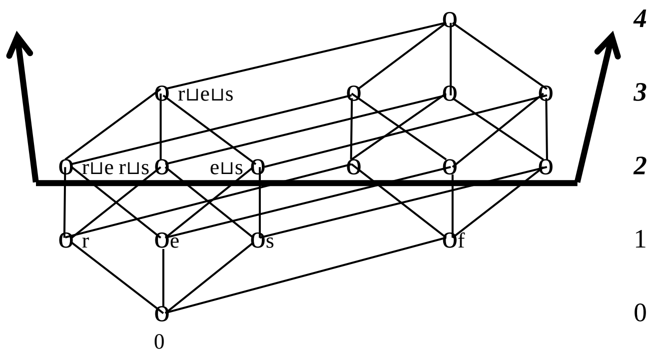
Exactly two $\rightarrow \lambda x.|x|=2$



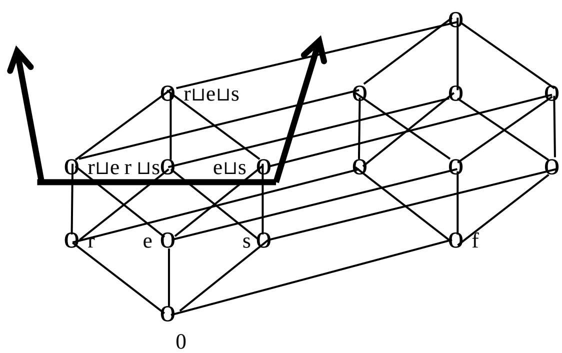
Exactly two cats $\rightarrow \lambda x.{}^*CAT_{w,t}(x) \wedge |x|=2$



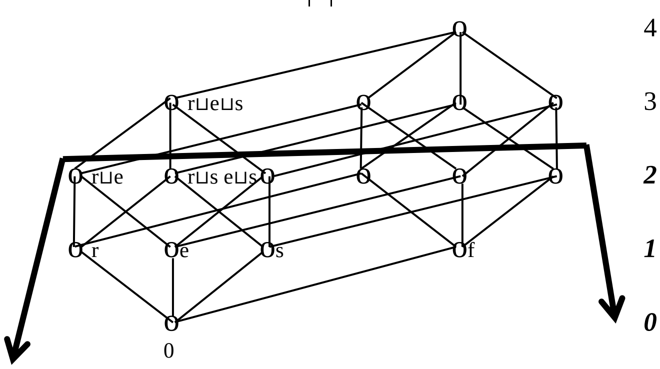
At least two $\rightarrow \lambda x.|x|\geq 2$



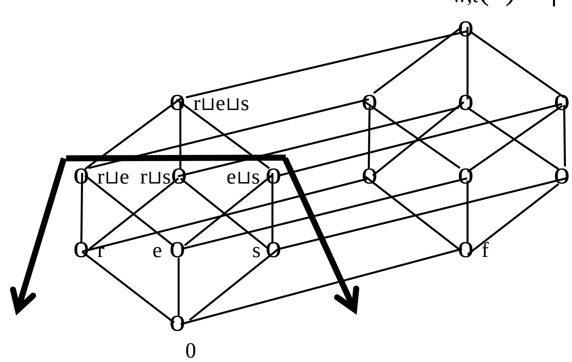
At least two cats $\rightarrow \lambda x.{}^*CAT_{w,t}(x) \wedge |x|\geq 2$



At most two $\rightarrow \lambda x.|x|\leq 2$



At most two cats $\rightarrow \lambda x.{}^*CAT_{w,t}(x) \wedge |x|\leq 2$



These pictures form a nice visual expression of how the polarity nature of the numerical DPs (downward entailing, upward entailing, neither up nor down) is directly determined by the number relation, ≤, ≥, = on the natural numbers, i.e. ≥ is closed downward on the natural numbers as indicated in the picture, ≤ is closed upward, and = is neither.

In Mountain semantics counting relates objects in the denotation of noun phrases to their atomic parts: they are counted in terms of the cardinality of the set of their atomic parts.

We are concerned with the shift CARD only in the context where the number phrase is an NP modifier (so we ignore here predicative cases like *The planets are seven*). In the modifier context, it is useful to analyze the role of the cardinality function more precisely, by making its presuppositional effect explicit. We do that most easily by letting CARD derive not a predicate at type <e,t>, but a *presuppositional intersective modifier* at type <<e,t>,<e,t>>:

Presuppositional cardinality shift:

$$\triangleright \text{CARD} = \lambda N\, \lambda P. \begin{cases} P \cap (N \circ \lambda x.|x|) & \text{if P is } \textit{count} \\ \bot & \text{otherwise} \end{cases}$$

We apply CARD to the number predicate interpretation of *at least three*:

at least three $\rightarrow \lambda n.n \geq 3$

and we get:

$$\text{CARD}(\lambda n.n \geq 3) = \lambda P. \begin{cases} \lambda x.P(x) \wedge |x| \geq 3 & \text{if P is } \textit{count} \\ \bot & \text{otherwise} \end{cases}$$

We now have a modifier interpretation for *at least three* which is undefined if it combines with a head NP whose interpretation is not count. When defined, as in the case of $*CAT_w$, the result of applying CARD($\lambda n.n \geq 3$) to $*CAT_w$ denotes, as before, the set of sums of cats with three atomic parts.

What we haven't defined here is what it means for a set of type <e,t> to be count. This can be defined in terms of atomicity:

Count sets: Let B be a complete Boolean algebra and $P \subseteq B$.
▹ P is *count* iff if $P \neq \emptyset$ and $P \neq \{0\}$ then $(P]$ is a complete *atomic* Boolean algebra.

If we want to use this to account for the felicity of number predicates with count nouns (✓*at least three cats*) and the infelicity of number predicates with mass nouns (#*at least three mud(s)*), we need define what it means for a *noun* to be a count noun. This can be done in terms of intensions:

Let $P: W \rightarrow \mathbf{pow}(B)$ be an intension.
▹ *Count intensions*: P is *count* iff for every $w \in W$: P_w is count.

And we make the obvious assumption:

Count noun phrases: Count noun phrases are interpreted as *count* intensions.

With this we can actually formulate an intensional version of the above **card** shifting rule:

Let *P* be a variable over intensions.

$$\textsc{card}(\lambda n.n \geq 3) = \lambda P.\begin{cases} \lambda x.P_w(x) \wedge |x| \geq 3 & \text{if } P \text{ is a } count \text{ intension} \\ \bot & \text{otherwise} \end{cases}$$

This means that we apply CARD(λn.n ≥ 3) to the intension of *cats*: $\lambda w.{*}CAT_w$.
This is indeed a count intension, hence we derive *at least three cats* with the standard interpretation.
If we apply CARD(λn.n ≥ 3) to the intension of *mud* , $\lambda w.MUD_w$, we assume that this intension is not count, and no felicitous interpretation is derived.

We see then that indeed in Mountain semantics counting makes reference to $ATOM_B$. Since we let the denotation of a singular predicate like *cat* be a set of atoms, the objects in this denotation, singular cats, are by definition objects of cardinality one. Objects in the plural denotation *cats* are counted in terms of their atomic parts.

Definite DPs

Definite article

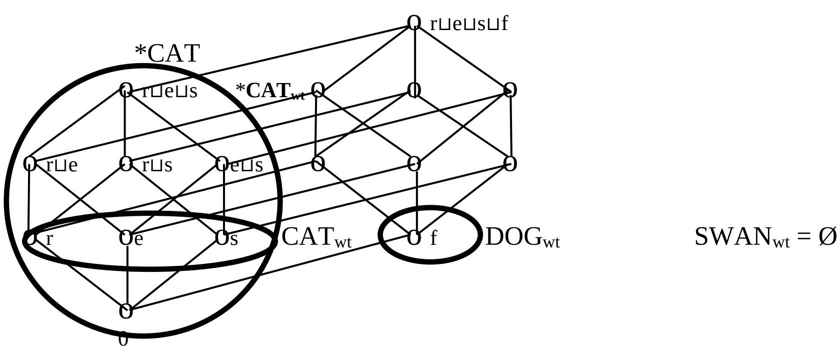
the → σ

$$\sigma = \lambda P.\begin{cases} \sqcup P & \text{if } \sqcup P \in P \\ \bot & \text{otherwise} \end{cases}$$

This analysis of the definite article was proposed in Sharvy 1980. It is not yet in Link 1983, but added in Link 1984.

Assume the following noun denotations:

cat → $CAT_{w,t}$ = {ronya, emma, shunra}
dog → $DOG_{w,t}$ = {fido}
swan → $SWAN_{wt}$ = Ø



the cat $\to \sigma(CAT_{wt}) = \sqcup(CAT_{wt})$ if $\sqcup(CAT_{wt}) \in CAT_{wt}$, $\bot$ otherwise
$\sqcup(CAT_{wt}) = r\sqcup e\sqcup s$ and $r\sqcup e\sqcup s \notin CAT_{wt}$,
hence $\sigma(CAT_{wt}) = \bot$
the cat is undefined in wt.

the cats $\to \sigma(*CAT_{wt}) = \sqcup(*CAT_{wt})$ if $\sqcup(*CAT_{wt}) \in *CAT_{wt}$, $\bot$ otherwise
$\sqcup(*CAT_{wt}) = r\sqcup e\sqcup s$ and $r\sqcup e\sqcup s \in *CAT_{wt}$,
hence $\sigma(*CAT_{wt}) = r\sqcup e\sqcup s$
the cats denotes $r\sqcup e\sqcup s$ in wt.

the dog $\to \sigma(DOG_{wt}) = \sqcup(DOG_{wt})$ if $\sqcup(DOG_{wt}) \in DOG_{wt}$, $\bot$ otherwise
$\sqcup(DOG_{wt}) = f$ and $f \in DOG_{wt}$,
hence $\sigma(DOG_{wt}) = f$
the dog denotes f in wt.

the swan $\to \sigma(SWAN_{wt}) = \sigma(\varnothing) = \sqcup(\varnothing)$ if $\sqcup(\varnothing) \in \varnothing$
the more than three swans $\to \sigma(\lambda x.*SWAN_{wt}(x) \wedge |x|>3) = \sigma(\varnothing) = \sqcup(\varnothing)$ if $\sqcup(\varnothing) \in \varnothing$
$\sqcup(\varnothing) = 0$ and $0 \notin \varnothing$,
hence $\sigma(\varnothing) = \bot$
the swan and *the more than three swans* are undefined in wt.

$*\varnothing = \{x: \exists X \subseteq \varnothing: x = \sqcup X\}$.
$\varnothing$ has exactly one subset, namely $\varnothing$. $\sqcup(\varnothing) = 0$.
Hence $*\varnothing = \{0\}$.
$\sqcup(*\varnothing) = \sqcup(\{0\}) = 0$ and $|0| = 0$

the swans $\to \sigma(*SWAN_{wt}) = \sqcup(*\varnothing)$ if $\sqcup(*\varnothing) \in *\varnothing$
$= 0$ if $0 \in \{0\}$
$= 0$
the less than three swans $\to \sigma(\lambda x.*SWAN_{wt}(x) \wedge |x|<3) =$
$\sqcup(\lambda x.x \in *\varnothing \wedge |x|<3)$ if $\sqcup(\lambda x.x \in *\varnothing \wedge |x|<3) \in \lambda x.x \in *\varnothing \wedge |x|<3) =$
$\sqcup(\lambda x.x \in \{0\} \wedge |x|<3)$ if $\sqcup(\lambda x.x \in \{0\} \wedge |x|<3) \in \lambda x.x \in \{0\} \wedge |x|<3) =$
0 if $0 \in \{0\}$
$= 0$
the swans and *the less than three swans* denote 0 in wt.

Using the null element
Frege has given us a semantics for the universal quantifier EVERY which makes EVERY(NP,VP) *trivially true* in w if the denotation of predicate NP in w is Ø. On Frege's analysis, asserting EVERY(NP,VP) may well have a *quantity implicature* that the denotation of this NP in w is not Ø, and even non-singleton, but this is not made part of the semantics. There are analyses of EVERY in the literature that differ from Frege's on this issue. For instance, de Jong and Verkuyl 1985 propose that EVERY *presupposes* that the denotation of the singular NP in EVERY(NP,VP) is non-empty, non-singleton.

Siding with Frege on this issue, I standardly use in introductory classes examples like (4) to argue, with Frege, that the non-emptiness, non-singleton effect here is an implicature and not a presupposition:

[I ran for some years a crackpot lottery and stand in court. I know that I shouldn't commit perjury. But I think I am better at Gricean pragmatics than the judge is, so I say:]

(4) a. Your honor, I swear that every person who, in the course of last year, presented me with a winning lottery ticket, has gotten his prize.
[I add, *sotte voce*, to you:]
b. Fortunately I was away all year on a polar expedition.

(4b) tells you (but not the judge) that the denotation of the NP *person who, in the course of last year, presented me with a winning lottery ticket* is empty. If the non-emptiness condition were a presupposition, the continuation (4b) should be infelicitous, because it directly contradicts the presupposion. But (4b) is not infelicitous, it cancels the non-emptiness implicature, and makes statement (4a) *trivially true*.

This means, of course, that (4a) violates the Gricean maxim of Quantity, because it doesn't give any information. But that is exactly my intention: I make a statement that is trivially true (no perjury), hoping that the judge (using standard Gricean reasoning) believes that it *is* true (Quality), but non-trivially so (Quantity). So I am trying to mislead the judge without making a false statement.

We now look at felicity versus triviality of definite DPs in the same courtroom context as in example (4). Imagine that I said instead of (4) one of the statements in (5):

(5) a. Your honor, I swear that *the one person* who, in the course of last year, presented me with a winning lottery ticket, has gotten his prize.
[*sotte voce*, to you:] b. #Fortunately I was away all year on a polar expedition.

b. Your honor, I swear that *the five persons* who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.
[*sotte voce*, to you:] b. #Fortunately I was away all year on a polar expedition.

c. Your honor, I swear that *the more than thirty persons* who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.
[*sotte voce*, to you:] b. #Fortunately I was away all year on a polar expedition.

In all these cases the continuation is infelicitous. Why? Because the examples in (5) *presuppose* that respectively one/ five/more than thirty persons came to me with a winning lottery ticket. And the continuation *denies* that. That is as good as a contradiction: the continuation conflicts with the presupposition.

We compare these cases with the cases in (6). Now imagine that I had said any of the statements in (6):

(6) a. Your honor, I swear that *the persons* who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.
[*sotte voce*, to you:] b. ✓Fortunately I was away all year on a polar expedition.
b. Your honor, the books ought to tell you how many people came to me last year to claim their prize. I am sure it was less than five. But I swear to you, your honor, that *the less than five persons* who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.
[*sotte voce*, to you:] b. ✓Fortunately I was away all year on a polar expedition.

The cases in (6) pattern with *every* in (4): the continuation is felicitous, which indicates that the non-emptiness claim is a cancellable implicature rather than a presupposition, and hence that the cases in (6) are quantity violations, rather than perjury.

Not all native English speakers that I have consulted are completely happy with DPs where the numerical is complex, in particular cases like *the at least ten persons who…* (although searching for cases like that on the web gives a surprisingly rich harvest). But even they agree that there is a *robust contrast* between the cases in (5) and in (6), and that is what is important here.

The cases in (5) are explained by the standard assumption concerning undefinedness and presupposition failure:

▻ *Definiteness:* If $\sigma(P_w) = \perp$ then $\varphi_w(\sigma(P_w))$ is *infelicitous*, due to presupposition failure.

The cases in (6) are explained by pragmatic manipulation of the null element:

▻ *Triviality:* If $\sigma(P_w) = 0$, then $\varphi_w(\sigma(P_w))$ is *trivial*, either trivially true or trivially false.

The assumption in Landman 2011b is that whereas the inclusion/exclusion of 0 in the denotation of NP denotations in languages like English is determined by and large by the compositional semantics, the inclusion/exclusion of 0 in the denotation of *verbal* predicates can be manipulated by pragmatics (just as semantic plurality for verbal predicates is not strictly fixed by compositional semantics: morphological number in the verbal domain is linked to agreement, not to semantic plurality, e.g. Landman 2000).

In the cases in (6) I make the statement $\varphi_w(\sigma(P_w))$ in front of the judge, where $\sigma(P_w) = 0$. The assumption that I *do* obey Quality leads to the assumption that $0 \in \varphi_w$, and if $0 \notin \varphi_w$ it leads to accommodating 0 in φ_w, shifting from φ_w to $\varphi_w \cup \{0\}$. With that the statement $\varphi_w(\sigma(P_w))$ is trivially true.

We find the same distinction between presupposition failure and triviality in examples like (7) and (8):

(7) a. In every family I know, the boys sleep together in one room, but *the one girl* has her own room.
b. In every family I know, the boys sleep together in one room, but *the two girls* have their own room.
c. In every family I know, the boys sleep together in one room, but *the more than three girls* have their own room.

If in one of the families I know there are no girls, all of (7a, b, c) are infelicitous. This is a consequence of the fact that the examples in (7) presuppose ($7a_p$, b_p, c_p):

(7) a_p In every family I know, there is one and only one girl.
b_p In every family I know, there are exactly two girls.
c_p In every family I know, there are more than three girls.

On the other hand, among the families concerned in the examples in (7), there could be families without boys, as long the respective presuppositions about the girls are satisfied. That is, the existence of a family without boys but with two girls in separate rooms does not make (7b) infelicitous, and in fact counts towards the truth conditions of (7b).

We now look at the examples in (8):

(8) a. In every family I know, the boys sleep together in one room, but *the girls* have their own room.
b. In every family I know, the boys sleep together in one room, but *the less than three girls* have their own room.

(8b) presupposes that in every family I know there aren't more than two girls (and admittedly a bit of context is required to make this a natural thing to say). But apart from that, the examples in (8) are *not* infelicitous if there is a family in which the boys sleep in one room *and there are no girls*. Thus, (8a) does neither presuppose that there are boys nor that there are girls in every family.

The semantics of the null object tells us that indeed the existence of such families is compatible with the truth conditions of the examples in (8): the universal quantifier can unproblematically quantify over a domain that contains them, because they are 'innocent': they do not contribute contingent information towards the truth conditions.

This can be seen as follows: the truth conditions of (8c) are given in ($8c_1$):

family $\rightarrow \text{FAMILY}_w \subseteq \text{ATOM}_{\mathbf{B}}$
girl $\rightarrow \text{GIRL}_w \subseteq \text{ATOM}_{\mathbf{B}}$
sleep in (her) own room $\rightarrow \text{SOR}_w \subseteq \text{ATOM}_{\mathbf{B}}$
$\mathbf{in}_w(x,f) \subseteq \text{ATOM}_{\mathbf{B}} \times \text{ATOM}_{\mathbf{B}}$

(8) c. In every family the girls sleep in their own room.
c_1. $\forall f[\text{FAMILY}_w(f) \rightarrow {}^*\text{SOR}_w(\sigma({}^*\lambda x.\text{GIRL}_w(x) \wedge \mathbf{in}_w(x,f_k)))]$

For our purposes here, it is easiest to think of ($8c_1$) as a conjunction ($8c_2$):

(8) c_2. $\varphi_1 \wedge \ldots \wedge \varphi_k \wedge \ldots \wedge \varphi_n$,
where $\varphi_k = (\text{FAMILY}_w(f_k) \wedge {}^*\text{SOR}_w(\sigma({}^*\lambda x.\text{GIRL}_w(x) \wedge \mathbf{in}_w(x,f_k)))]$

φ_k means: f_k is a family and each girl in f_k sleeps in her own room.

Take φ_k and assume: $\lambda x.\text{GIRL}_w(x) \wedge \mathbf{in}_w(x,f_k) = \emptyset$.
Then: ${}^*\lambda x.\text{GIRL}_w(x) \wedge \mathbf{in}_w(x,f_k) = \{0\}$.
Then: $\sigma({}^*\lambda x.\text{GIRL}_w(x) \wedge \mathbf{in}_w(x,f_k)) = 0$

Then $*SOR_w(\sigma(\lambda x.GIRL_w(x) \wedge \mathbf{in}_w(x,f_k)))$ is trivially true, because $0 \in *SOR_w$

But this means that φ_k drops out of the conjunction:

(8c3) $\varphi_1 \wedge \ldots \wedge \varphi_{k-1} \wedge \boldsymbol{\varphi_k} \wedge \varphi_{i+k} \wedge \ldots \wedge \varphi_n$ =
$\varphi_1 \wedge \ldots \wedge \varphi_{k-1} \wedge$ **1** $\wedge \varphi_{k+1} \wedge \ldots \wedge \varphi_n$ =
$\varphi_1 \wedge \ldots \wedge \varphi_{k-1} \wedge \quad \varphi_{k+1} \wedge \ldots \wedge \varphi_n$

In other words, Mountain semantics with the unrestricted semantic plurality operation * predicts that if *girl in family* f_k denotes Ø, then *the girls in family* f_k denotes 0, and hence that family f_k is *irrelevant* for the truth of (8c), because *the girls in family* f_k *sleep in their own room* is trivially true.

And this means that we can unproblematically assume that the universal quantifier in (8c) quantifies over all families I know, including the ones without girls, since the truth conditions of (8c) don't depend on the latter families.

I conclude that the theory of semantic plurality encoded in the operation * turns out to have very interesting linguistic bite: it predicts distinctions between definite DPs whose denotation suffers from presupposition failure and definite DPs whose denotation is trivial, and these distinctions show up in the linguistic data.

The distributive operator

(6) a. *The three cats* eat half a can of tuna.
b. *The three cats* eat half a can of tuna *each*.

In (6a) it is undetermined whether the cats eat half a can of tuna together, or whether each of them eats that much tuna. The latter reading is made explicit in (6b) with distributor *each*. Link 1983 analyzes (6b) by assuming that it involves the same predicate *eat half a can of tuna* as (6a), and *each* is interpreted as a distributive operator ^{D} that operates at the VP level:

$^D = \lambda P \lambda x.\forall a \in ATOM_x: P(a)$

Fact: For all $P \subseteq B$: $^DP = *ATOM_{\sqcup P}$

cat $\rightarrow CAT_{wt}$ with $CAT_{wt} = \{r, e, s\} \subseteq ATOM_{\mathbf{B}}$
the three cats $\rightarrow \sigma(\lambda x.*CAT_{wt}(x) \wedge |x|=3) = r \sqcup e \sqcup s$

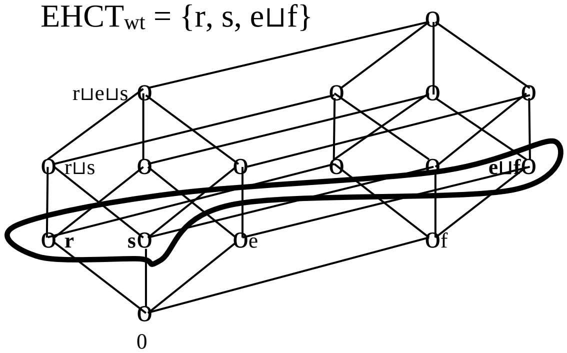
eat half a can of tuna $\rightarrow EHCT_{wt} \subseteq B$

eat half a can of tune each $\rightarrow {}^DEHCT_{wt}$ =
$\lambda x.\forall a \in ATOM_x: EHCT_{wt}(a)$
(6b) $\rightarrow \lambda x.\forall a \in ATOM_x: EHCT_{wt}(a)(r \sqcup e \sqcup s)$ =
$\forall a \in ATOM_{r \sqcup e \sqcup s}: EHCT_{wt}(a)$ =
$EHCT_{wt}(r) \wedge EHCT_{wt}(e) \wedge EHCT_{wt}(s)$

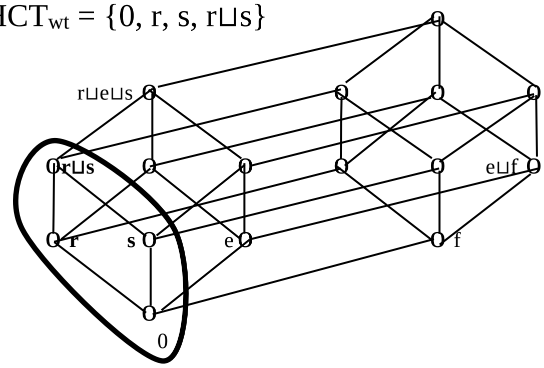
Example: If in wt Ronya eats half a can of tuna and Shunra eats half a can of tuna and Emma and Fido share half a can of tuna, then: $EHCT_{wt} = \{r, s, e \sqcup f\}$
In that case, $^{D}EHCT_{wt} = \{0, r, s, r \sqcup s\}$
and $r \sqcup e \sqcup s \notin {}^{D}EHCT_{wt}$.

The set indicated in the left structure is the set of all objects in B that eat half a can of tuna. The set indicated in the right structure is the set of all objects in B of which all the atomic parts eat half a can of tuna.

$EHCT_{wt} = \{r, s, e \sqcup f\}$



$^{D}EHCT_{wt} = \{0, r, s, r \sqcup s\}$



(6a) is false in wt, because $r \sqcup e \sqcup s \notin \{0,r,s,r \sqcup s\}$

(7) *The two white cats* eat half a can of tuna each.

Let:
white cat $\rightarrow \lambda x.CAT_{wt}(x) \wedge WHITE_{wt}(x) = \{r,s\}$
the two white cats $\rightarrow \sigma(^{*}\lambda x.CAT_{wt}(x) \wedge WHITE_{wt}(x)) = r \sqcup s$
(7) is true in wt, because $r \sqcup s \in \{0,r,s,r \sqcup s\}$
We see that the semantics of distributive adverbial *each*, and more generally the distributive operator, make reference to atoms. Hence, these three phenomena - counting, cardinality comparison, and distributivity - have often been regarded as diagnostics for count nouns.

We can make the ^{D} operator similarly presuppositional:

$$^{D}P_{wt} = \lambda x. \begin{cases} \forall a \in ATOM_{x}: P_{wt}(a) & \text{if } (x] \text{ is } \textit{count} \\ \bot & \text{otherwise} \end{cases}$$

Example:
Let *special* $\rightarrow SPECIAL_{wt}$

$$^{D}SPECIAL_{wt} = \lambda x. \begin{cases} \forall a \in ATOM_{x}: SPECIAL_{wt}(a) & \text{if } (x] \text{ is } \textit{count} \\ \bot & \text{otherwise} \end{cases}$$

(8) a. The cats are each special.
b. #The wine is each special.

We assume that *cat* $\rightarrow$ $CAT_{wt} \subseteq ATOM_{\mathbf{B}}$ and that *wine* $\rightarrow$ $WINE_{wt}$ and that at wt $WINE_{wt}$ is not count. Hence $(\sqcup(CAT_{wt})]$ is a complete atomic Boolean algebra and $(\sqcup(WINE_{wt})]$ is not.

$^{D}SPECIAL_{wt}(\sigma(*CAT_{wt}))$ is defined, since $(\sqcup(CAT_{wt})]$ is count.
$^{D}SPECIAL_{wt}(\sigma(WINE_{wt}))$ is undefined, since $(\sqcup(WINE_{wt})]$ is not count.

4.3 Boolean semantics for mass nouns and count nouns

Link 1983 proposes a semantics in which mass and count nouns are interpreted in distinct but linked domains. I give here a version close to that in Landman 1991:

▹ A *Boolean interpretation domain* is a structure $\boldsymbol{B} = \langle \mathbf{B},\mathbf{M},\mathbf{C}, \downarrow,\uparrow \rangle$, where:

1. **B** is a complete Boolean algebra such that $B = {}^{*}(M \cup C)$.
2. **M**, *the mass domain*, is a complete Boolean algebra.
3. **C**, *the count domain*, is a complete atomic Boolean algebra.
4. $0_{\mathbf{B}} = 0_{\mathbf{M}} = 0_{\mathbf{C}}$.
5. $\downarrow$, *the grinding function*, is a function from C into M such that:
   for every $x \in C$: $\downarrow x = \sqcup_{\mathbf{M}}\{\downarrow a: a \in ATOM_x\}$.

   Grinding maps a count element onto the sum of its mass parts.
6. $\uparrow$, *the portioning function*, is a one-one function from M into $ATOM_{\mathbf{C}} \cup \{0\}$ such that:
   $\uparrow(0) = 0$ and for every $x \in M$: $\downarrow(\uparrow(x)) = x$.

   Portioning treats a mass object as an atomic count object.

Lemma: $\downarrow$ is a join-homomorphism, a homomorphism that preserves 0, $\sqsubseteq$ and $\sqcup$.

Proof:
1. $\downarrow(0_C) = \sqcup_{\mathbf{M}} (\{\downarrow a: a \in ATOM_{0_C}\}) = \sqcup_{\mathbf{M}} (\emptyset) = 0_{\mathbf{M}}$
2. If $x \sqsubseteq_{\mathbf{C}} y$, then $ATOM_x \subseteq ATOM_y$.
Then $\{\downarrow a: a \in ATOM_x\} \subseteq \{\downarrow a: a \in ATOM_y\}$, and hence $\sqcup_{\mathbf{M}}\{\downarrow a: a \in ATOM_x\} \sqsubseteq_{\mathbf{M}} \sqcup_{\mathbf{M}}\{\downarrow a: a \in ATOM_y\}$,
so $\downarrow(x) \sqsubseteq_{\mathbf{M}} \downarrow(y)$.
3. Let $X \subseteq C$. We need to show that $\downarrow(\sqcup_{\mathbf{C}}(X)) = \sqcup_{\mathbf{M}}(\{\downarrow(x): x \in X\})$

By definition:	[1] $\downarrow(\sqcup_{\mathbf{C}}(X)) = \sqcup_{\mathbf{M}}(\{\downarrow(a): a \in ATOM_{\sqcup_{\mathbf{C}}(X)}\})$
By atomisticity:	[2] $ATOM_{\sqcup_{\mathbf{C}}(X)} = \bigcup_{x \in X} ATOM_x$
Hence:	[3] $\{\downarrow(a): a \in ATOM_{\sqcup_{\mathbf{C}}(X)}\} = \bigcup_{x \in X}(\{\downarrow(a): a \in ATOM_x\})$
And hence:	[4] $\sqcup_{\mathbf{M}}(\{\downarrow(a): a \in ATOM_{\sqcup_{\mathbf{C}}(X)}\}) = \sqcup_{\mathbf{M}}(\bigcup_{x \in X}(\{\downarrow(a): a \in ATOM_x\}))$
So we have derived:	[5] $\downarrow(\sqcup_{\mathbf{C}}(X)) = \sqcup_{\mathbf{M}}(\bigcup_{x \in X}(\{\downarrow(a): a \in ATOM_x\}))$
But it is easy to see that (by join):	[6] $\sqcup_{\mathbf{M}}(\bigcup_{x \in X}(\{\downarrow(a): a \in ATOM_x\})) = \sqcup_{\mathbf{M}}(\{\downarrow(x): x \in X\})$
Hence:	[7] $\downarrow(\sqcup_{\mathbf{C}}(X)) = \sqcup_{\mathbf{M}}(\{\downarrow(x): x \in X\})$

▹ A *Boolean interpretation structure* is a structure $\boldsymbol{I} = \langle \boldsymbol{B},W \rangle$, where $\boldsymbol{B}$ is a Boolean interpretation domain and W is a set of indices.

Intensions are functions P: W $\rightarrow$ **pow**(B)

Two useful notions are *cumulativity* and *homogeneity*:

Let $X \subseteq B$
▹ X is *cumulative* iff if $X \neq Ø$ then $X = {}^*X$.
▹ X is *homogenous* iff if $X \neq Ø$ then $X = (\sqcup X]$.

A set is cumulative if it is identical to its own closure under sum. A set is homogenous if it is identical to its own part set.

Lemma: If X is homogenous, then X is cumulative.
If $X \subseteq ATOM_B$, then *X is homogenous

Proof:
1. If X is homogenous, then $X = (\sqcup X]$. $(\sqcup X]$ is closed under $\sqcup$. Namely, let $Y \subseteq (\sqcup X]$, then for every $y \in Y$: $y \sqsubseteq \sqcup X$. Hence $\sqcup Y \sqsubseteq \sqcup X$, hence $\sqcup Y \in (\sqcup X]$. So $^*(\sqcup X] = (\sqcup X]$. Hence X is closed under $\sqcup$, and hence $X = {}^*X$.
2. Let $X \subseteq ATOM_B$. Then $ATOM_{\sqcup X} = X$. $\sqcup X$ is the maximum of *X. Let $y \sqsubseteq \sqcup X$. Since $y = \sqcup ATOM_y$, it follows $\sqcup ATOM_y \sqsubseteq \sqcup ATOM_{\sqcup X}$, and hence $ATOM_y \subseteq ATOM_x$, and so $ATOM_y \subseteq X$. Hence $y \in {}^*X$.
This means that $^*X = (\sqcup X]$. Hence *X is homogenous.

Let P: $W \rightarrow$ **pow**(B) be an intension.
▹ P is *cumulative* iff for every $w \in W$: P_w is cumulative.
▹ P is *homogenous* iff for every $w \in W$: P_w is homogenous.

Lønning 1987 gives an analysis in which *lexical mass nouns* are interpreted as *homogenous* intensions. Krifka 1986, 1989 proposes that lexical mass nouns be interpreted as *cumulative intensions*. In both cases the semantics explores analogies between the denotations of lexical mass nouns and *lexical plural nouns*. For Lønning both are homogenous, for Krifka both are cumulative.

Example:

Lexical mass noun: *wine* $\rightarrow$ $WINE_w$, where $WINE_w$ is a cumulative subset of M .

$WINE_M$ is cumulative means that $WINE_w = {}^*WINE_w$. Hence it follows that $\sqcup WINE_w \in WINE_w$. And this means that σ is defined for $WINE_w$:

the wine $\rightarrow \sigma(WINE_w) = \sqcup WINE_w$ The sum of the wine

Cumulativity is shown for plurals in (15) and for mass nouns in (16):

Let *cat* $\rightarrow$ CAT_w *pet of mine* $\rightarrow$ $MY\text{-}PET_w$ *pet of yours* $\rightarrow$ $YOUR\text{-}PET_w$
Marc $\rightarrow$ $MARC_w$ *liquid in my glass* $\rightarrow$ $MY\text{-}LIQUID_w$ *liquid in my glass* $\rightarrow$ $YOUR\text{-}LIQUID_w$
where CAT_w, $MY\text{-}PET_w$, $YOUR\text{-}PET_w \subseteq ATOM_C$
and $MARC_w$, $MY\text{-}LIQUID_w$, $YOUR\text{-}LIQUID_w$ are cumulative subsets of M.

(15b) and (16b) are provably valid inferences:

(15) a. If my pets are cats and your pets are cats then my pets and your pets are cats.
b. $*CAT_w(\sigma(*MY\text{-}PET_w)) \wedge *CAT_w(\sigma(*YOUR\text{-}PET_w)) \Rightarrow$
$*CAT_w(\ \sigma(*MY\text{-}PET_w) \sqcup \sigma(*YOUR\text{-}PET_w)\ )$

(16) a. If the liquid in my glass is Marc de Bourgogne and the liquid in your glass is Marc de Bourgogne, then the liquid in my glass and the liquid in your glass is Marc de Bourgogne.
b. $MARC_w(\sigma(MY\text{-}LIQUID_w)) \wedge MARC_w(\sigma(YOUR\text{-}LIQUID_w)) \Rightarrow$
$MARC_w(\ \sigma(MY\text{-}LIQUID_w) \sqcup \sigma(YOUR\text{-}LIQUID_w)\ )$

Two special cases of Boolean interpretation domains are discussed in the literature, the Boosk domain and the Classical domain:

The Boosk domain [Link 1983]
▹ A *boosk* is a Boolean interpretation domain where $\uparrow$ is the identity function.

For Link 1983 all mass objects are themselves atomic count objects (so the mass partial order $\sqsubseteq_{\mathbf{M}}$ is an order between objects that are themselves atoms with respect to the count partial order $\sqsubseteq_{\mathbf{C}}$).

The classical domain [Landman 1991]
▹ A *classical domain* is a Boolean interpretation domain where M is atomless and $C^+ \cap M^+ = \emptyset$.

If the mass domain is atomless and the denotation MUD_w of mass noun *mud* is homogenous, then MUD_w is *divisible*:

▹ X is *divisible* iff $\forall x \in X^+ \exists x_1 \in X^+ \exists x_2 \in X^+[x = x_1 \sqcup x_2 \wedge x_1 \sqcap x_2 = 0]$.

If MUD_w is divisible, then every object that counts as mud can be partitioned into two objects that also count as mud. Count noun denotations cannot satisfy divisibility, since divisibility stops at atoms. The idea that mass noun denotations differ from count noun denotation in that mass noun denotations satisfy some sort of divisibility and hence are not built from minimal elements or don't have minimal elements at all, was common in the earlier ('classical') semantic literature (e.g. ter Meulen 1980, Bunt 1985, Link 1983, Landman 1991). A representative example is given by the following (almost) quote:

> "What are the minimal parts of water? Chemistry tells us that they are the water molecules. But water molecules can be counted, while water cannot be counted. This shows that natural language semantics does not incorporate the insights of chemistry in its models: in our semantic domains, the water molecules are not the minimal parts of water. In fact, the real semantic question is: is there any evidence, semantic evidence, to assume that mass entities like water are built from minimal parts at all, either from minimal parts that are water, or from minimal parts that aren't water? If there is no such semantic evidence, it is theoretically better to assume that the semantic system does not impose a requirement of minimal parts.
> Since there is no semantic evidence for minimal parts, we should assume non-atomic structures for the mass domain. That has the added bonus that we can nicely explain why we cannot count mass entities, because counting is counting of atoms." (paraphrase of Landman 1991, pp 312-313)

We defined earlier:

▻ *Count sets*: Z is *count* iff if $Z^+ \neq Ø$ then (Z] is a complete *atomic* Boolean algebra.
▻ *Count intensions*: P is *count* iff for every $w \in W$: P_w is count.
▻ *Count NPs*: NP α is *count* iff for every Boolean interpretation structure and interpretation function, α is interpreted as a *count* intension.

For lexical count nouns, like *cat*, *count* takes the form of a stipulation:

Lexical constraint: Interpret lexical count nouns as count intensions.

Given this, for complex noun phrases like *cats* or *three blind mice* you don't have to stipulate that they are count: you can actually *prove* that they are.

For mass nouns there are different options.

▻ *Z* is *non-atomic* iff if $Z^+ \neq Ø$ then (Z] is a complete *non-atomic* Boolean algebra.
▻ *Z* is *atomless* iff if $Z^+ \neq Ø$ then (Z] is a complete *atomless* Boolean algebra.

And we can identify mass with the first or the second notion:

▻ Z is *non-atomic-mass* iff Z is non-atomic.
▻ Z is *atomless-mass* iff Z is atomless.

And this generalizes to intensions and noun phrases:

▻ *Mass intensions*: P is *mass* iff for every $w \in W$: P_w is mass.
▻ *Mass NPs*: NP α is *mass* iff for every Boolean interpretation structure and interpretation function, α is interpreted as a *mass* intension.

Lexical constraint: Interpret lexical mass nouns as mass intensions.

Lemma: In a classical Boolean interpretation structure:
Intensions in $(W \to \mathbf{pow}(C))$ are count.
Intensions in $(W \to \mathbf{pow}(M))$ are atomless mass.
Intensions in $(W \to \mathbf{pow}(B))$ that are neither in $(W \to \mathbf{pow}(C))$ nor in $(W \to \mathbf{pow}(M))$ are non-atomic-mass.

Proof: Let ***B*** be a classical interpretation domain.
-If $Z \subseteq C$ and $Z^+ \neq Ø$ then (Z] is a complete atomic Boolean algebra.
-If $Z \subseteq M$ and $Z^+ \neq Ø$ then (Z] is a complete atomless Boolean algebra.
-If $Z^+ \cap M \neq Ø$ and $Z^+ \cap C \neq Ø$ and $Z^+ \neq Ø$ then (Z] is a complete non-atomic Boolean algebra which is neither atomic nor atomless.

We look at counting, count comparison and distribution in classical interpretation domains. The classical theory most directly fits the idea that these three phenomena be regarded as *diagnostics* for count nouns.

1. *Counting:* Count nouns can be modified felicitously by numerical phrases, mass nouns cannot.

(17) a. ✓one fish   /✓two fish
b. # one mud / # two mud

2. *Count comparison:* Count nouns only get a *count comparison* readings with *most*, mass nouns only get *measure readings* with *most*.

(18) a. Most cats eat a can of tuna a day.

If Ronya and Emma each weigh a quarter of what Shunra weights, and Ronya and Emma each eat a quarter can of tuna a day, while Shunra eats a can of tuna a day, it is not the case that (18a) is true because Shunra's weight (and volume) is greater than the combined weight (and volume) or Ronya and Emma. (18a) is false, because only one out of three cats eat a can of tuna a day.

(18) b. Most mud was deposited as a blanket of sediment that settled slowly out of suspension [γ]

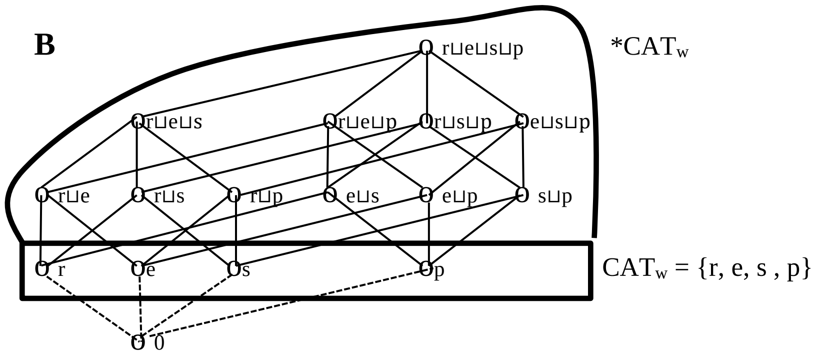
3. Distributivity: Count DPs combine with distributive predicates, mass DPs do not.

(19) a. ✓The cats have each eaten a can of tuna.
b. #The mud has each sunk to the bottom.

The Classical theory presents a picture of the mass-count distinction that is very crisp and clear. Too crisp and too clear: the above diagnostics are not in fact secure as diagnostics for the mass/count distinction. This leads to Iceberg Semantics.

4.4. Iceberg semantics

The name *Mountain semantics* was motivated by the visual image of the denotation of the plural noun *cats* as rising up like a mountain from the atomic seabed, i.e. from the denotation of the singular noun *cat* as a set of atoms: (where p = PIM).

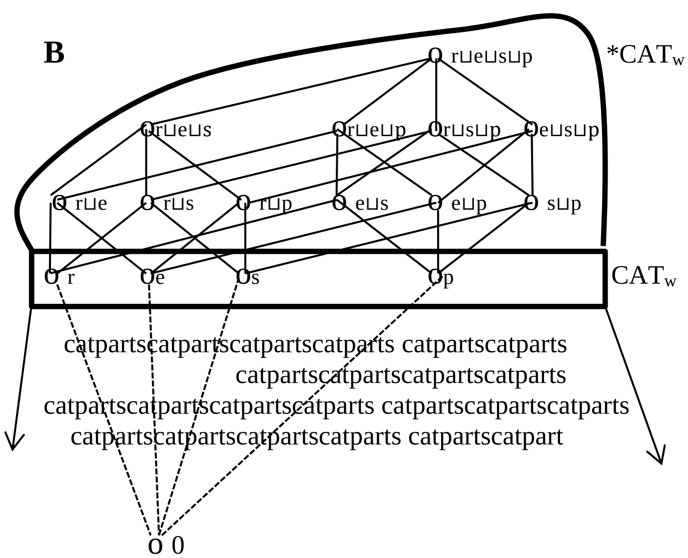


Iceberg semantics replaces the condition that the denotation of the singular noun *cat* be a set of atoms, by the weaker condition that it be a *disjoint set*. This removes the *sorting* that Mountain Semantics assumes: mass and count denotations can live in the same Boolean domain and be ordered by the same Boolean operations (since count noun denotations are no longer assumed to be atomic). To get the correct Boolean structure for plural count noun denotation, we use the theorem we mentioned in the section above:

Theorem: if X is a *disjoint subset* of B, then *X forms a complete atomic Boolean algebra.

If we assume that count noun denotations are (in context) disjoint, the plural denotation is indeed guaranteed to have the correct Boolean structure.

This means that the denotation of the plural noun *cats* is still visually like a mountain, the same mountain, rising up from the denotation of the singular noun *cat*, but now the latter is only a disjoint set. This means that the mountain is lifted off the atomic seabed, and hence floats in a sea of cat parts, like an *Iceberg*:



But, in order to preserve the attractive features of Mountain semantics: we need to keep track in the compositional semantics of the distribution set.

We do that by enriching the semantics. An iceberg is going to be a *pair* of sets *X* = <**body**(X), **base**(*X*)>, where the *body* of the iceberg is in essence the interpretation that we got in Mountain semantics, and where the *base* of the iceberg is a set that generates the **body** under sum.

In the case of count nouns, the **base** is the set in terms of which elements in the body of the interpretation of the count noun are counted, count compared and to which distribution takes place. Where in Mountain semantics the grammar assigns a set denotation to a complex NP based on the interpretations of the parts, Iceberg semantics assigns an Iceberg, a <**body**, **base**> pair as denotation to a complex NP, based on the interpretations of the parts, and in the process of building up the **base** of the interpretation of the complex NP, the semantics can keep track of the distribution set.

Let **B** be a complete Boolean algebra.

▻ An *i-set* is a pair *X* = <**body**(*X*), **base**(*X*)> where:

body(*X*) $\subseteq$ B and **base**(*X*) $\subseteq$ B and **body**(*X*) $\subseteq$ ***base**(*X*) and $\sqcup$**body**(*X*) = $\sqcup$**base**(*X*).

An i-set is a pair consisting of a **body** set and a **base** set, where the base generates the body under sum.

▻ An *i-object* is a pair *x* = <**body**(*x*), **base**(*x*)> where:

body(*x*) $\in$ B and **base**(*x*) $\subseteq$ B and **body**(*x*) = $\sqcup$(**base**(*x*))

An i-object is a pair consisting of a **body** object and a **base** set, where the **base** generates the body under sum (**body** = $\sqcup$**base**).

In Iceberg semantics NPs are interpreted as i-sets and definite DPs are interpreted as i-objects. The semantics in a nutshell:

Singular count nouns: *cat* $\rightarrow$ < CAT_w, CAT_w >, where CAT_w is a *disjoint* set
Plural count nouns: *cats* $\rightarrow$ < $*CAT_w$, CAT_w >
Singular definite DPs: *the cat* $\rightarrow$ < $\sigma(CAT_w)$, CAT_w >
Plural definite DPs: *the cats* $\rightarrow$ < $\sigma(*CAT_w)$, CAT_w >

Idea: we choose a disjoint set CAT_w = {r, s, e, p}.
The singular noun *cat* denotes the i-set <CAT_w, CAT_w> = <{r, s, e, p}, {r, s, e, p}>, with the same **body** and **base**.
The plural noun *cats* has the same **base** as the singular noun, but its **body** is, as in Mountain semantics, the closure under sum.
The definite DP *the cats* denotes the i-object <$r\sqcup e\sqcup s\sqcup p$, {r,e,s,p}>.
The **body** is the object that we assumed in Mountain semantics, the **base** is the distribution set:

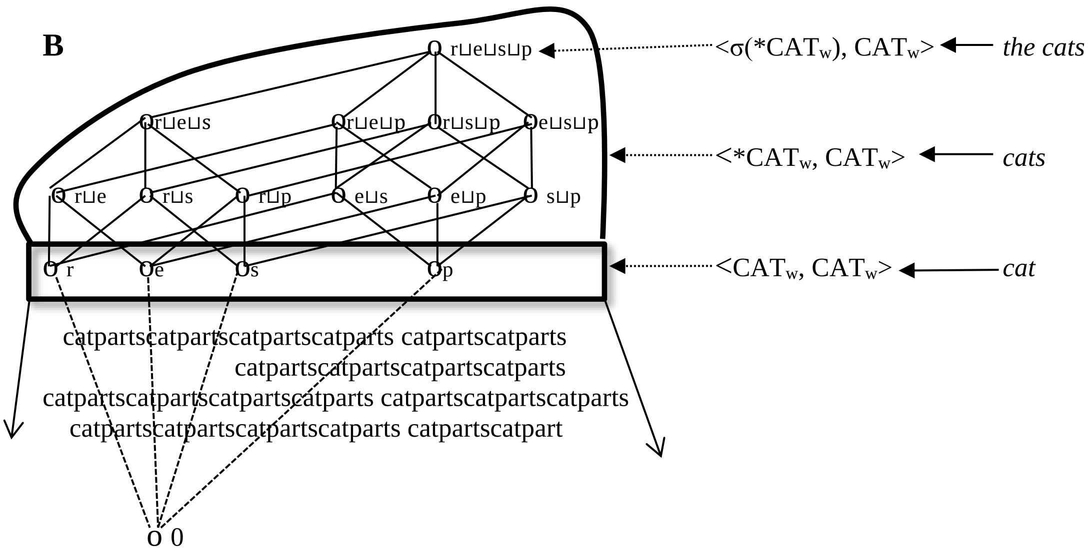
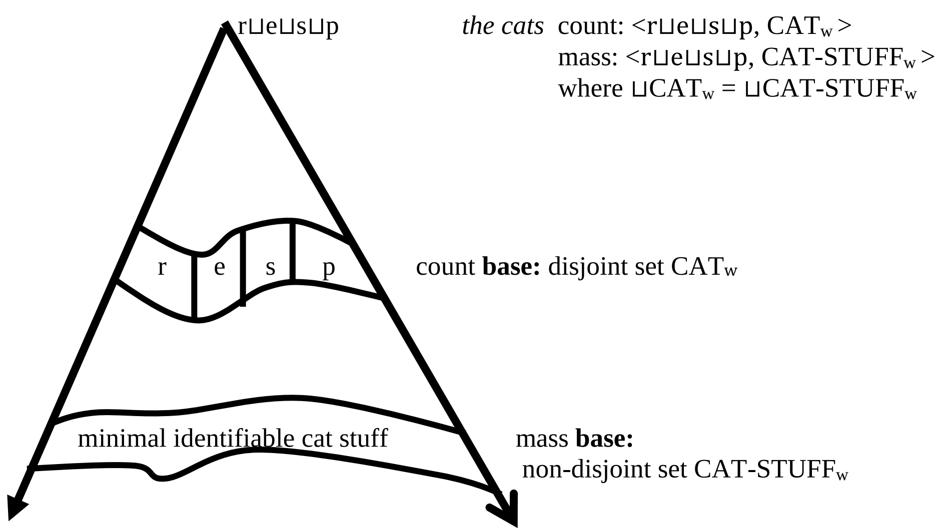


Figure 4.3

This theory explicitly allows for the possibility that *the same sum*, r⊔e⊔s⊔p, the denotation of *the cats*, can be regarded as a count object or as a mass object depending on the **base**. The theory is not sorted: the cat stuff making up Ronya can be regarded as part of Ronya in the sense of $\sqsubseteq_B$. In a picture:



Since the semantics no longer accesses the atoms of **B** (if indeed there are any), and since the notion of cardinality in Mountain semantics is defined in terms of $ATOM_{\mathbf{B}}$ we need a different notion of cardinality in Iceberg semantics. We introduce this notion by defining the notion of distribution set:

Presuppositional distribution set $\mathbf{D}_Z(x)$:

$$\triangleright\ \mathbf{D} = \lambda Z \lambda x. \begin{cases} (x] \cap Z & \text{if } Z \text{ is } \textit{disjoint} \\ \bot & \text{otherwise} \end{cases}$$

.

$\mathbf{D}_Z(x)$, the distribution set of x relative to Z, is the set of Z-parts of x, *presupposing that* Z *is disjoint.* $\mathbf{D}_Z(x)$ is the set of Z-objects in terms of which x is counted, and to which a distributive predicate distributes, when it is applied to x.

With this, we define the *presuppositional cardinality* of x relative to set Z: $\mathbf{card}_Z(x)$:

$$\triangleright\ \mathbf{card} = \lambda Z \lambda x. \begin{cases} |\mathbf{D}_Z(x)| & \text{if } Z \text{ is } \textit{disjoint} \\ \bot & \text{otherwise} \end{cases}$$

.

If Z is disjoint, then $\mathbf{card}_Z(x) = |\mathbf{D}_Z(x)| = |(x] \cap Z|$, the cardinality of the set of Z-parts of x.

Take the above example:

cats $\rightarrow$ $<{}^*CAT_w, CAT_w>$ with $CAT_w = \{r, e, s, p\}$, a disjoint set.

$r \sqcup e \sqcup s \in {}^*CAT_w$.

Since CAT_w is a disjoint set, $\mathbf{card}_{CAT_w}(r \sqcup e \sqcup s)$ is defined, and we calculate:

$\mathbf{card}_{CAT_w}(r \sqcup e \sqcup s) = |\mathbf{D}_{\{r,\, e,\, s,\, p\}}(r \sqcup e \sqcup s)| = |(r \sqcup e \sqcup s] \cap \{r, e, s, p\}|$

Since $\{r, e, s, p\}$ is disjoint, $p \notin (r \sqcup e \sqcup s]$. Hence $|(r \sqcup e \sqcup s] \cap \{r, e, s, p\}| = |\{r, e, s\}| = 3$.

Thus indeed, if we calculate the cardinality of sum $r \sqcup e \sqcup s$ relative to set CAT_w - which means that we take the elements of CAT_w *in this context* to be the elements that count as one - then $r \sqcup e \sqcup s$ counts as three, relative to that set.

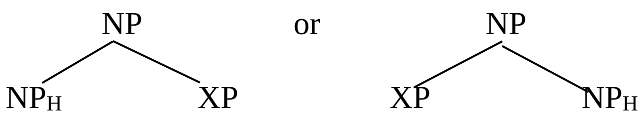
Alternatively, if we calculate $\mathbf{card}_{\mathbf{CAT-STUFF_w}}(r \sqcup e \sqcup s)$, we get:

$$\mathbf{card}_{\mathbf{CAT-STUFF_w}}(r \sqcup e \sqcup s) = \bot$$

This is because we took $CAT\text{-}STUFF_w$ to be a set that is not disjoint, hence the presupposition of **card** fails.

Compositionality and the Head principle
Iceberg semantics: the **base** information of the interpretation of complex expressions is derived compositionally from the base information of the interpretations of the parts.

The Head principle for NPs:
Take a complex NP with head NP_H and interpretations as given:



$NP \rightarrow \alpha$ where $\alpha = <\mathbf{body}(\alpha), \mathbf{base}(\alpha)>$
$NP_H \rightarrow H_\alpha$ where $H_\alpha = <\mathbf{body}(H_\alpha), \mathbf{base}(H_\alpha)>$

The Head principle tells us that **base**(α) is determined by **body**(α) and **base**(H_α) in the following way:

▻ *Head principle for* NPs: $\mathbf{base}(\alpha) = (\!|\mathbf{body}(\alpha)|\!) \cap \mathbf{base}(H_\alpha)$

So the **base** of the interpretation of a complex NP is determined by the **body** of the interpretation of that complex NP and the **base** of the interpretation of the *head* of that complex NP. The *Head principle* specifies that base information is *passed up* from the interpretation of the head of a complex NP to the interpretation of that complex NP: the **base** of the interpretation of the complex NP is the set of all Boolean parts of the **body** of the interpretation of the complex NP *intersected with* the **base** of the interpretation of the head.

Lemma: If **base**(H_α) is disjoint then **base**(α) is disjoint.

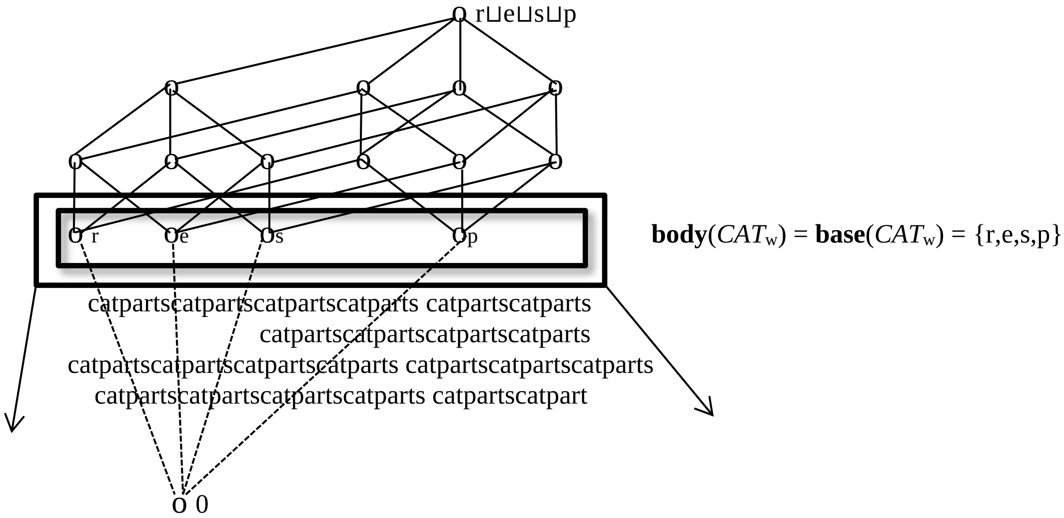
Proof: $\mathbf{base}(\alpha) \subseteq \mathbf{base}(H_\alpha)$.

The corollary says that, if the **base** of the interpretation of the head of an NP is disjoint, then the **base** of the interpretation of the NP itself is also disjoint. If we follow the spirit of the move from Mountain semantics to Iceberg semantics, and replace the characterization of *count* in terms of atomicity by a characterization of *count* in terms of *disjointness* (as we will do in the next chapter) it will follow from the Head principle that a complex NP with a count NP as head, is itself a count NP.

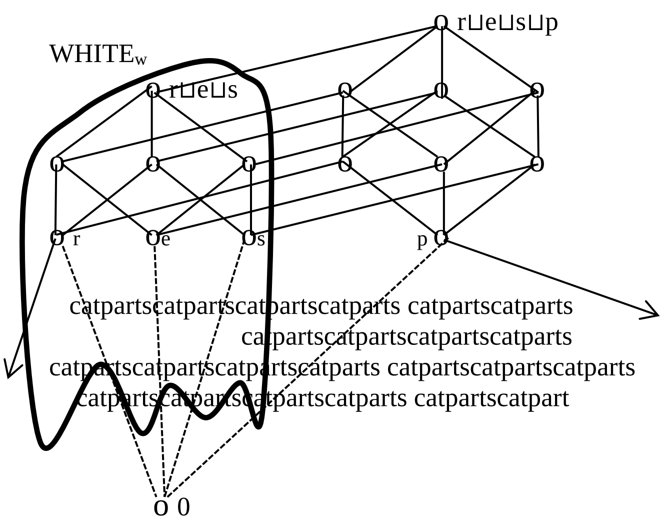
This means that the Head principle allows us to formulate a compositional semantic theory of the notions *mass* and *count* (and also the notions *neat* and *mess* from Landman 2011a): the mass-count characteristics of the head NP inherit up to the complex NP.

Example: *the three white cats*

Step 1: *cat* → CAT_w = <CAT_w, CAT_w> with CAT_w = {r, e, s, p}, a disjoint set:



Step 2: We assume the set of white objects to be the set $WHITE_w$:



Step 3: We interpret *white cat*.

white → λP.< **body**(P) ∩ $WHITE_w$, **(body**(P) ∩ $WHITE_w$**]** ∩ **base**(P) >

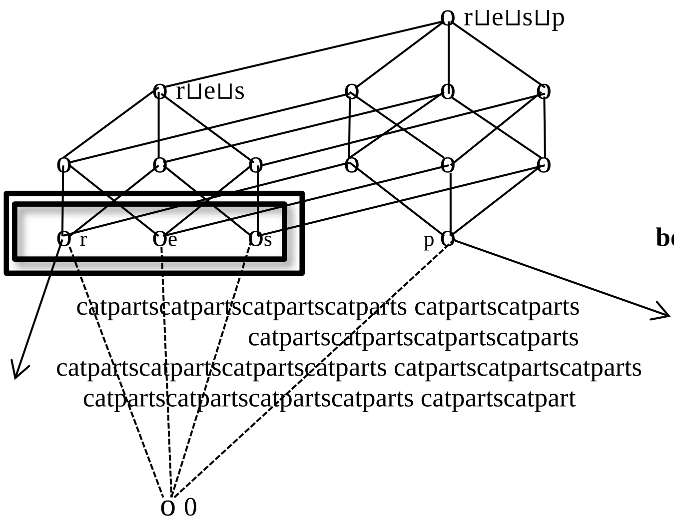
Hence:

white cat → $WHITE\ CAT_w$ =

  λP.< **body**(P) ∩ $WHITE_w$, **(body**(P) ∩ $WHITE_w$**]** ∩ **base**(P)> (<CAT_w, CAT_w>)

  = <CAT_w ∩ $WHITE_w$, **(**CAT_w ∩ $WHITE_w$**]** ∩ CAT_w>

  = <CAT_w ∩ $WHITE_w$, CAT_w ∩ $WHITE_w$>



$\mathbf{body}(\mathit{WHITE\ CAT}_w) = \mathbf{base}(\mathit{WHITE\ CAT}_w) = \{r, e, s\}$

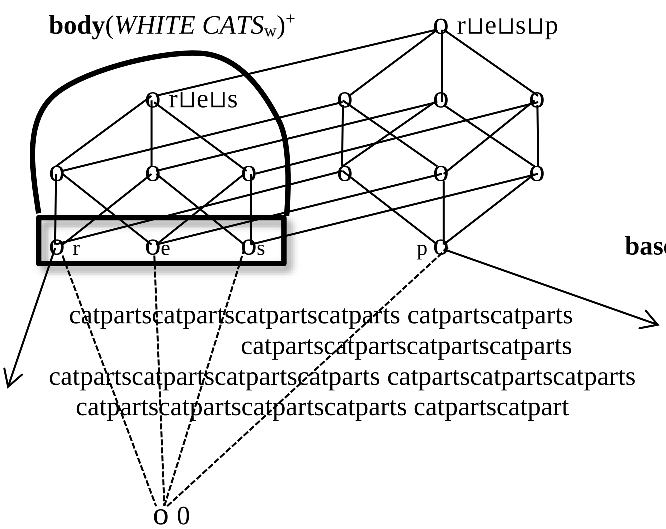
Step 4: We now pluralize.

$\triangleright\ \mathbf{plur} = \lambda P.\langle {}^*\mathbf{body}(P), \boldsymbol{(}{}^*\mathbf{body}(P)\boldsymbol{]} \cap \mathbf{base}(P)\rangle$

$\mathbf{plur}(\mathit{WHITE\ CAT}_w) = \mathit{WHITE\ CATS}_w =$

$\qquad \lambda P.\langle {}^*\mathbf{body}(P), \boldsymbol{(}{}^*\mathbf{body}(P)\boldsymbol{]} \cap \mathbf{base}(P)\rangle\ (\langle \mathrm{CAT}_w \cap \mathrm{WHITE}_w, \mathrm{CAT}_w \cap \mathrm{WHITE}_w\rangle)$

$\qquad = \langle {}^*(\mathrm{CAT}_w \cap \mathrm{WHITE}_w), \boldsymbol{(}{}^*(\mathrm{CAT}_w \cap \mathrm{WHITE}_w)\boldsymbol{]} \cap (\mathrm{CAT}_w \cap \mathrm{WHITE}_w)\rangle$



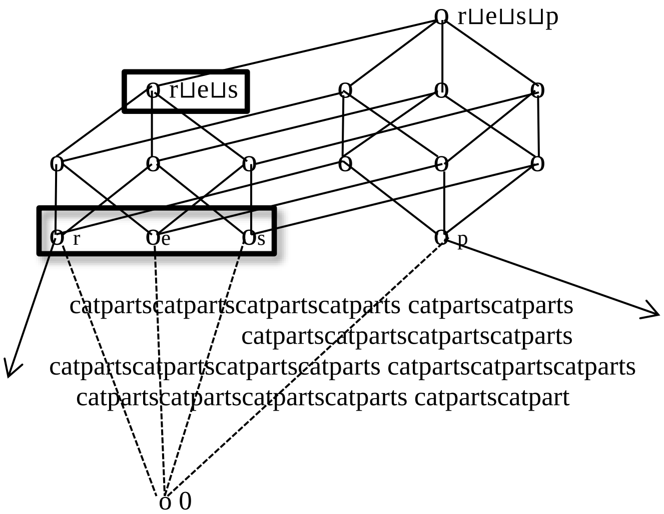
$\mathbf{base}(\mathit{WHITE\ CATS}_w) = \{r, e, s\}$

Step 5: *three white cats*

$$\mathit{three} \rightarrow \lambda P. \begin{cases} \langle \lambda x.\ \mathbf{body}(P)(x) \wedge \mathbf{card}_{\mathbf{base}(P)}(x) = 3, \\ \quad \boldsymbol{(}\lambda x.\ \mathbf{body}(P)(x) \wedge \mathbf{card}_{\mathbf{base}(P)}(x)\boldsymbol{]} \cap \mathbf{base}(P)\rangle & \text{if } \mathbf{base}(P) \text{ is disjoint} \\ \bot & \text{otherwise} \end{cases}$$

three white cats $\rightarrow$ *THREE WHITE CATS*$_w$ =

$$< \lambda x.\ {}^*(CAT_w \cap WHITE_w)(x) \wedge \mathbf{card}_{CAT_w \cap WHITE_w}(x)=3,$$
$$\mathbf{(}\lambda x.\ {}^*(CAT_w \cap WHITE_w)(x) \wedge \mathbf{card}_{CAT_w \cap WHITE_w}(x)=3\mathbf{]} \cap (CAT_w \cap WHITE_w) >$$
$$= < \lambda x.\ {}^*(CAT_w \cap WHITE_w)(x) \wedge \mathbf{card}_{CAT_w \cap WHITE_w}(x)=3,\ \ CAT_w \cap WHITE_w >$$



body(*THREE WHITE CATS*$_w$) = {r⊔e⊔s}

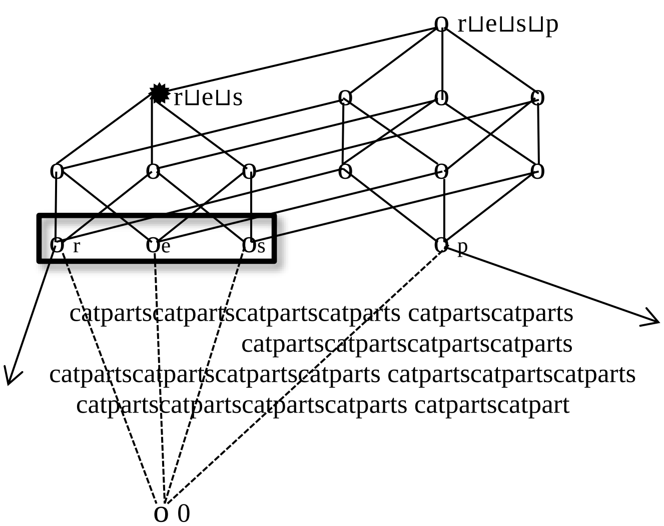
base(*THREE WHITE CATS*$_w$) = {r, e, s}

Step 6: *The three white cats*

▹ *the* $\rightarrow \lambda P.\ <\sigma(\mathbf{body}(P)),\ \mathbf{(}\sigma(\mathbf{body}(P))\mathbf{]} \cap \mathbf{base}(P)>$

the three white cats $\rightarrow$ *THE THREE WHITE CATS*$_w$ =

$$<\sigma(\lambda x.{}^*(CAT_w \wedge WHITE_w)(x) \wedge \mathbf{card}_{CAT_w \cap WHITE_w}(x)=3),\ CAT_w \cap WHITE_w >$$
$$= <\{r\sqcup e\sqcup s\},\ \{r, e, s\}>$$



body(*THE THREE WHITE CATS*$_w$) = r⊔e⊔s

base(*THE THREE WHITE CATS*$_w$) = {r, e, s}

4.5. Iceberg semantics for neat mass nouns (*livestock, poultry, furniture, pottery*…)

▻ *X* is *count* iff **base**(*X*) is disjoint. For i-set *X*
▻ *X* is *mass* iff (if *X* is non-null then) *X* is *not count*.

▻ *X* is *neat* iff **base**(*X*) is atomistic and $\text{ATOM}_{\mathbf{base}(X)}$ is disjoint.
▻ *X* is *mess* iff (if *X* is non-null then) *X* is *not neat*.

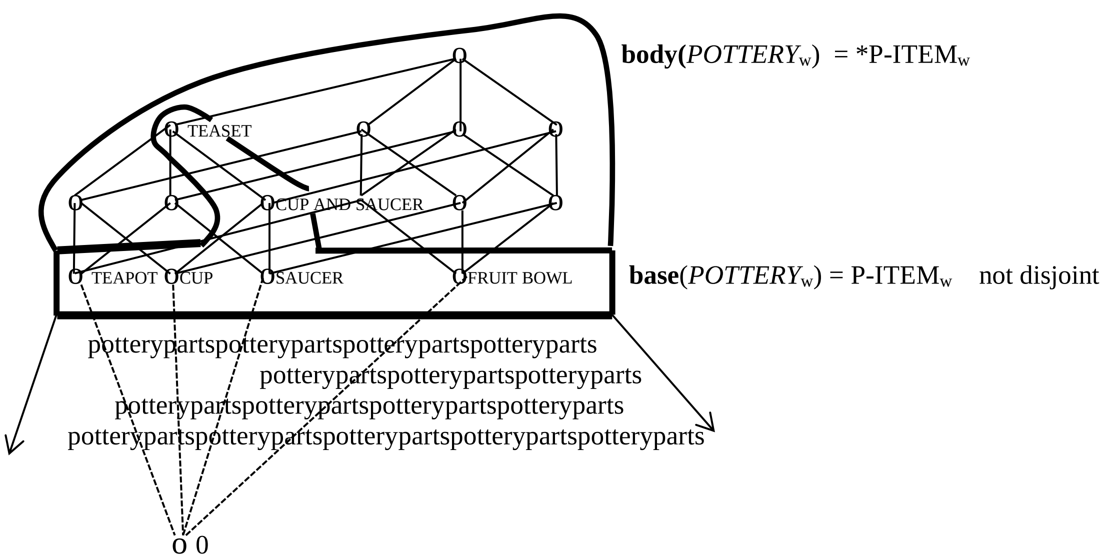
Group-neutral neat mass nouns
▻ The i-set denotation of a neat mass noun is *group neutral* if the distinction between individuals and groups, *aggregates*, conglomerates of individuals, is neutralized in the **base**.

-Count nouns keep individuals in the **base** and groups of such base individuals separate:
a group of cats is not itself a cat.
-Neat mass nouns like *pottery* do not adhere to that distinction: a group of pottery items can count in the right context as *one* wrt. the denotation of *pottery*, ***alongside*** its parts that also count as *one*.

Example: *pottery*
So: in our shop you can buy *cups* and *saucers* independently, but you can also buy a *cup and saucer* (for a different price), and you can but a one-person *teaset* for a very good price. But a *saucer and fruit bowl* is not an item sold as one in our shop.

*Set of pottery items **building blocks***:
P-ATOM_w = {THE TEAPOT, THE CUP, THE SAUCER, THE FRUIT BOWL}, **a disjoint set**.

Set of pottery items sold as one:
P-ITM_w = {THE TEAPOT, THE CUP, THE SAUCER, THE FRUIT BOWL, THE CUP AND SAUCER, THE TEASET}, **not disjoint**
$POTTERY_w$ = <**body**($POTTERY_w$), **base**($POTTERY_w$)>
where **body**($POTTERY_w$) = $^*\text{P-ITEM}_w$ and **base**($POTTERY_w$) = P-ITEM_w



Fact: *POTTERY*$_w$ is a neat mass i-set.

Sum-neutral neat mass nouns

▻ The i-set denotation of a neat mass noun is *sum neutral* if the distinction between the **base** and the **body** is neutralized.

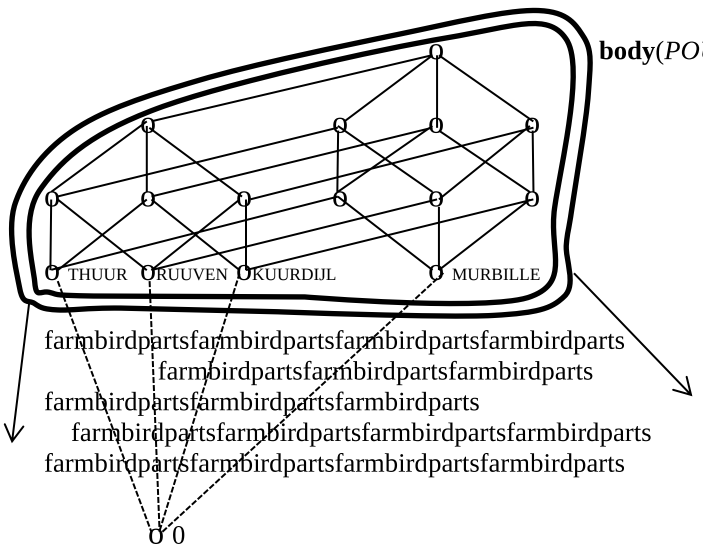
▻ *X* is *sum neutral* iff for some disjoint set $X \subseteq B$: $X = \langle *X, *X \rangle$

Natural cases that are sum neutral are mass nouns for natural kinds, like *livestock* and *poultry*:

Example: *poultry*
Assume that in w we are at a turkey farm, and all the relevant farm birds are turkeys.

$\text{ATOM}_{\mathbf{base}(POULTRY_w)}$ = FARM BIRD$_w$= {THUUR, RUUVEN, KUURDIJL, MURBILLE}, a disjoint set.

farm bird → *FARM-BIRD*$_w$ = < FARM BIRD$_w$, FARM BIRD$_w$ >
poultry → *POULTRY*$_w$ = < *FARM BIRD$_w$, *FARM BIRD$_w$ >



***FARM BIRD$_w$* is a singular count i-set.**
***POULTRY*$_w$ is a *sum neutral neat mass* i-set.**

Within neat denotations, *plural count* (<*X,X>) and *sum neutral* (<*X,*X>) are the extreme cases. *Group neutrality* is an in-between case.
Sum neutral neat mass nouns allow count comparison only with respect to the set of **base** atoms.
Group neutral neat mass nouns allow contextual variation.

Count comparison and measure comparison for neat mass nouns
Barner and Snedeker 2005: neat mass nouns, like count nouns, allow count-comparison interpretations:

(1) a. Most *farm animals* are outside in summer.
b. Most *livestock* is outside in summer.

Example: On our neighbor's farm there is large livestock: 10 cows, weighing all together 700 kg.,
and *poultry* (feathered livestock): 100 chickens, weighing all together 60 kg.
On this farm, the chickens are inside all year through, but the cows are outside in summer.
Both (1a) and (1b) allow a reading on which what is asserted is false = count comparison

Bunt 1982, 2005 (following Quite 1960): Widespread assumption about neat mass nouns: Neat mass nouns are semantically no different from count nouns.
The only difference is that neat mass nouns grammatically lack a feature +COUNT.
Against this: Rothstein 2011, Landman 2011, Grimm and Levin 2012:
Neat mass nouns are semantically different from count nouns in that they, unlike count nouns, allow measure comparison interpretations: e.g. example (2):

(2) a. ✓Although more farm animals are inside than outside, as concerns biomass, *most livestock* is outside in summer. Also in terms of volume, *most livestock* is outside.
b. **#** Although more livestock is inside than outside, as concerns biomass, *most farm animals* are outside in summer. Also in terms of volume, *most farm animals* are outside.

Count nouns do not allow measure comparison with *most*,
Hence: neat mass nouns do differ *semantically* from count nouns.

Semantic idea: count nouns and neat mass noun allow comparison in terms of a disjoint set,
the set of **base** atoms in case of count nouns (= the **base**) and sum-neutral neat mass nouns,
a contextual provided disjoint set of **base** elements in the case of group-neutral neat mass nouns.

4.6. Types of mess mass nouns

▻ *X* is *mess mass* iff *X* is *mass* and *X* is *mess*
iff **base**(*X*) is not disjoint and *either* **base**(*X*) is not atomistic
or **base**(*X*) is atomistic but $\mathbf{ATOM}_{\mathbf{base}(X)}$ is not disjoint.

The disjunctive definition of mess mass i-sets allows a range of different types of i-sets that all count as mess mass, from completely homogenous i-sets to heterogeneous i-sets.
I discuss three types here.

TYPE 1: HOMOGENEOUS i-SETS: example: ***time***

Mess mass noun *time* as in (7):

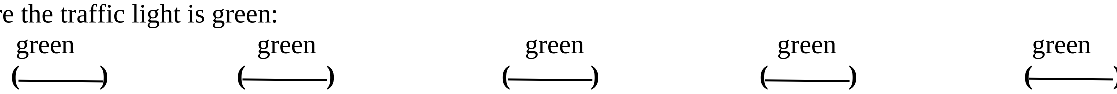
(7) Much *time* had passed.

time → $TIME_w$ = <**body**($TIME_w$), **base**($TIME_w$)>

body($TIME_w$): set of periods of time.

Period structure of time: $\mathbb{P}$ set of regular open subsets of $\mathbb{R}$

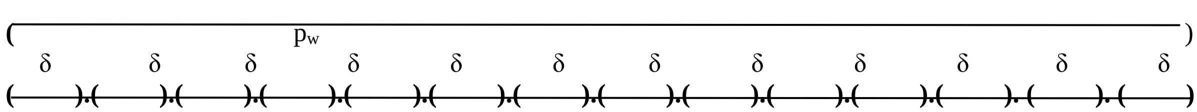
The notion of a period is a generalization of the notion of an open interval. Example: the picture shows the period where the traffic light is green:

p = 

- $p_w \in \mathbb{P}$ is the contextually maximal period in **body**($TIME_w$), and (for ease) an interval.
- **duration** is a measure function from open subintervals of p_w to $\mathbb{R}$.
- δ be a contextually given number in $\mathbb{R}$, such that we cannot in the context distinguish between intervals of size r and subintervals of smaller sizes.

- *Moments of time* in p_w*:* M_{p_w} is a set of open sub-intervals intervals of size δ that *partitions* p_w.

Fact: $\cup M_{p_w}$ is a set with single points missing between its maximal subintervals:



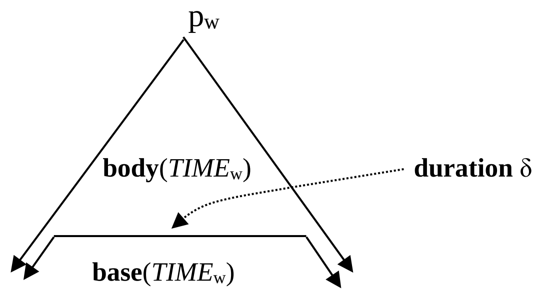
body($TIME_w$) = $(p_w]$ the set of all subperiods of p_w

What is **base**($TIME_w$)? Here is a suggestion.
base($TIME_w$) = $\{p \in (p_w]^+ : \mathbf{duration}(p) \leq \delta\}$
the set of subperiods with duration up to δ

So we get:

time → $TIME_w$ = < **(**p_w**]**, {p ∈ **(**p_w**]**$^+$: **duration**(p) ≤ δ} >

base($TIME_w$) forms the *bottom* of the **body**($TIME_w$):



Fact 1: The interpretation of *time, TIME*$_w$ is a *mess mass* i-set.

Let *moment of time* → M_{p_w} = < $\mathrm{M}_{\mathrm{p_w}}$, $\mathrm{M}_{\mathrm{p_w}}$>

Fact 2: The interpretation of *moment of time* M_{p_w} is a *singular count* i-set.

Homogeneous mess mass: it's *time* all the way down.

TYPE 2: CONTEXTUALLY CHOSEN OVERLAPPING MINIMAL PARTS: example: ***meat***

Landman 2011 (paraphrase): Take a big juicy slab of meat. We can think of this as being built from minimal parts, without having to assume that there are 'natural minimal meat parts'; think of the meat as built from parts that are appropriately minimal in the context. For instance, they are the pieces as small as a skilled butcher, or our special fine-grained meat-cutting machine can cut them. Suppose the meat cutting machine consists of a horizontal sheet knife and a vertical lattice knife that cut the meat into tiny cubes: snap – snap. This will partition the meat into many tiny meat cubes, which we can see as contextual minimal parts.

Now, if we move the sheet-knife or the lattice-knife a little bit, we get a *different* partition of the meat into minimal meat cubes. And there are many ways of moving the sheet knife and the lattice knife, each giving a different partition. None of these partitions has a privileged status (as providing 'natural' or ‘real’ minimal parts); the meat can be seen as built from all of them. This provides an i-set that is mess mass.

Boolean structure of *regions of space*: $\mathbb{I}$, set of regular open subsets of $\mathbb{R}^3$.
π_w: B → $\mathbb{I}$ maps objects onto the region of space they occupy (eigenplace).
We take again a top down perspective: Let m_w be the sum of the meat in w.
The meat cutter would, with the current position of its blades, cut m_w into a ***variant***, a set of parts of m_w that are little cubes.

▹ a *variant* for m_w is a set $\mathbf{var}_{m_{w},\delta}$ which satisfies the conditions $V_1 – V_5$:
V_1. A variant is a partition of the meat m_w
V_2. A variant also partitions the space of the meat m_w
V_3. The variant cuts the meat into little blocks
V_4. The little blocks have the same volume
V_{5a}. Each block in the variant is the maximal part of the meat occupying the space of that block
V_{5b}. Contextual volume value δ is *big enough* so that we recognize the *maximal parts of* m_w that go on at the regions of volume δ as *meat*: contextual mimimal parts that are meat.

▹ $\mathbf{V}_{m_{w,\delta}}$ is the set of all variants for m_w.

▹ $MEAT_w$ is the union of all the meat variants: $MEAT_w = \cup \mathbf{V}_{m_{w},\delta}$

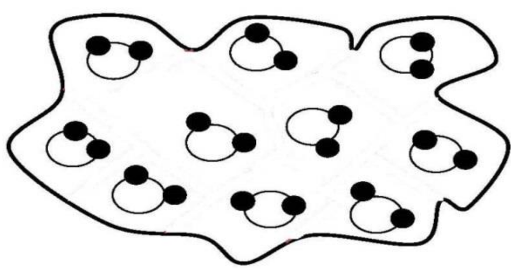
meat → MEAT_w = <*$MEAT_w$, $MEAT_w$>

We take as the **base** of the i-set MEAT_w the union of the variants, and as **body** the closure of this set under sum.

Fact: $\boldsymbol{\mathit{MEAT}_w}$ is a mess mass i-set.

TYPE 3: HETEROGENEOUS I-SETS: example: ***water***

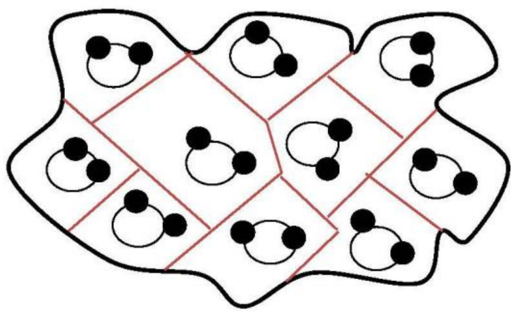
Landman 2011: (Paraphrase) Here is a puddle of water. Look down into the water of the puddle:



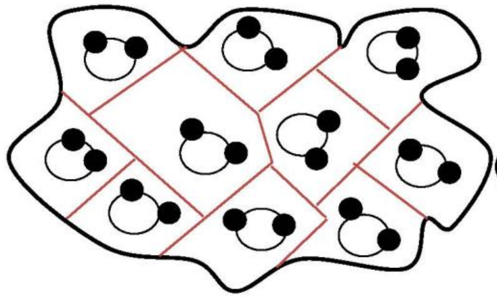
Count perspective: The water is built from a disjoint set of water molecules. There is only one variant. Hence it is reasonable to regard the water as just the sum of the water molecules.
Mass perspective: The puddle as a spatio-temporal object: when you look down into the puddle, you don't just see a set of water molecules, you see these objects in their spatio-temporal configurations and the relations between them. More in particular, you see what is a conglomeration of objects in space. When you divide up what you see in front of you, you cannot pick and choose: you're dividing up the puddle into sets of water molecules *and space.*

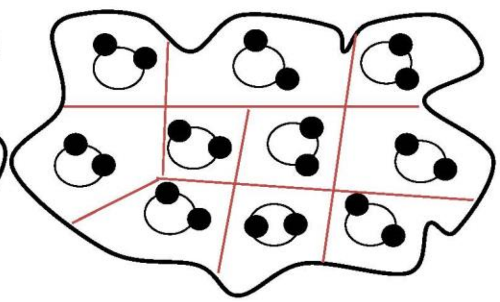
So you *can*, if you so want, pick the cherries out of the pie, pick the disjoint individual molecules out of the space, but that is imposing a count perspective. On the mass perspective, you pick the molecules out, by dividing the puddle into a disjoint set of water molecule-space pairs, which means that you simultaneously divide up the set of molecules and the space they are in.

Spatio temporal count perspective: It is perfectly reasonable to regard the puddle as the sum of disjoint building blocks. say, blocks that have exactly one water molecule in them, blocks that partition the sum of water molecules *and* its space:



But, again, such partitions are not unique, they are variants, and on the mass perspective such a variant does not have a special status:





So, even though the set of water molecules would not itself give rise to an overlapping **base**, water molecules *cum* space, do.

Idea: body(*WATER*$_w$) consists of sums of water molecules plus regions of space containing these, making up in total the water molecules in the puddle and the space of the puddle.

base(*WATER*$_w$) is a set of water molecule-space pairs that *that contain a single water molecule.*

Intuition: a subregion of the water that contains one water molecule may well counts itself as *water*, but a subregion that only contains, say, half a molecule does not itself count as *water*.

We assume that all the (contextually relevant) water in w is the water making up the puddle.

E_w is the set of all water molecules in w, $e_w = \sqcup E_w$ and $\mathbf{e}_w = \langle e_w, \pi_w(e_w)\rangle$.

▹ We construct **base**(*WATER*$_w$) and **body**(*WATER*$_w$) as sets of pairs <e,π>, where e is a sum of water molecules and π is a region that $\pi_w(e)$ is a *proper part* of, i.e. $\pi_w(e) \subset \pi$

▹ A variant for $\mathbf{e}_w$ is a set $\mathbf{var}_{e_w}$ is a set of molecule-space pairs <e,π> as described above where $\mathbf{dom}(\mathbf{var}_{e_w})$ is a partition of e_w and $\mathbf{ran}(\mathbf{var}_{e_w})$ is a partition of $\pi_w(e_w)$.

▹ $\mathbf{V}_{e_w}$ is the set of all variants of water.

As before, we let the **base** of *WATER*$_w$ be the union of variants:

water → *WATER*$_w$ = <*WATER$_w$, WATER$_w$>, where WATER$_w$ = $\cup\mathbf{V}_{e_w}$

Fact: base($WATER_w$) is an atomless mess mass i-set.

THE SUPREMUM ARGUMENT

Two different choices for the interpretation of the NP *water molecule*:

Interpretation 1: the base of *water molecule* is a variant in the base of *water*:
water molecule $\rightarrow WM_w = \langle WM_w, WM_w \rangle$, where $WM_w \in \mathbf{V}_{e_w}$

Interpretation 1*: Water molecule* denote a variant of *water*, a partition of the water and its space in w

Fact: $\boldsymbol{WM_w}$ **is a singular count i-set such that** $\sqcup$**body(**$\boldsymbol{WM_w}$**) =** $\sqcup$**body(**$\boldsymbol{WATER_w}$**)**

-On interpretation 1, the mass DP *the water* and the count DP *the water molecules* have the same body-denotation: the mass supremum and the count supremum are identified.

Interpretation 2: the base of *water molecule* is a set of water molecules:
water molecule $\rightarrow E_w = \langle E_w, E_w \rangle$, where E_w is disjoint.

Interpretation 2: We ignore the spatio-temporal setting of the water, and fish the molecules out of the space, treat them as abstract objects on their own merit, and distance them in that way from the denotation of *the water*.

Fact: $\boldsymbol{E_w}$ **is a singular count i-set such that:** $\sqcup$**body(**$\boldsymbol{E_w}$**)** $\neq$ $\sqcup$**body(**$\boldsymbol{WATER_w}$**)**

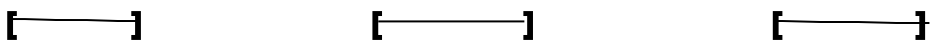
-On interpretation 2, the mass DP *the water* and the count DP *the water molecules do not* have the same body-denotation: the mass supremum and the count supremum are not identified.

Thus, Iceberg semantics does not have to take a stand on Chierchia 1998's Supremum Argument (in favor of interpretation 1). Iceberg semantics can allow both perspectives.
Landman 2019b argues that this is a Good Thing.

4.7. The standard model of regular open sets and separateness

In topology, the notion of regular open set as defined in chapter 4 is generalized to a notion of regular open set in a topological space. The idea of the general definition is quite simple: Let the *closure of* x be the result of adding the bounds (infimums and supremums) to x:

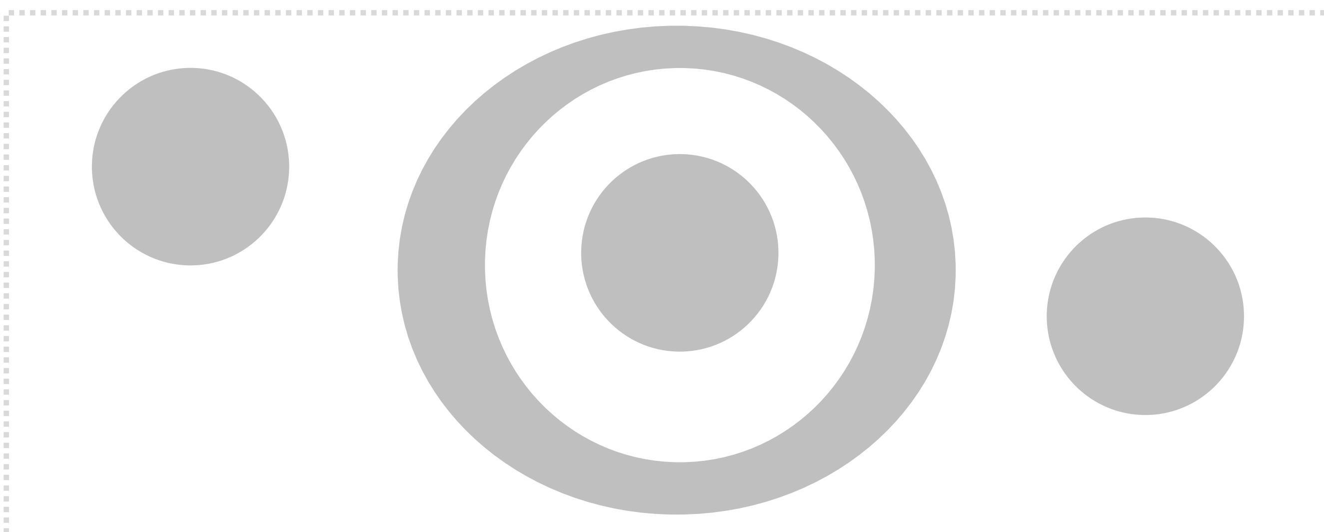
x



The *interior of the closure of* x consists of the points *between* the bounds, this is the operation ~~x. If x is a regular open set, it is identical to the interior of its closure (x = ~~x). We have seen that if x is an open set but not regular set, ~~x adds points, filling up the cracks. The topological strategy is to *define* a regular open set as the interior of its closure.

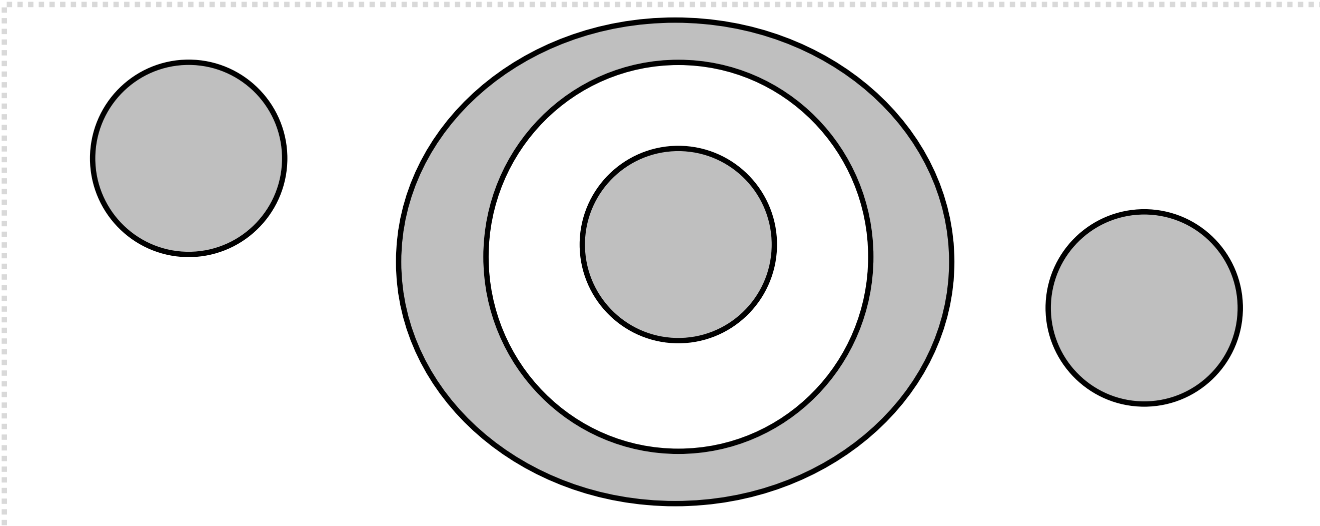
When we come to models for space we need to generalized these notions to regular open sets of points in space, $\mathbb{R}^2$, $\mathbb{R}^3$,...

Looking briefly at two-dimensional space, an example of a regular open set in two-dimensional space could be the set of grey points in the following picture:



x

Here too we need to define a notions of closed and open bounds, where, as we can see, bounds are no longer single points, but curves. Intuitively, the bounds separate what is inside the regular open set (grey) from what is outside (white), which means that adding the bounds will give us:



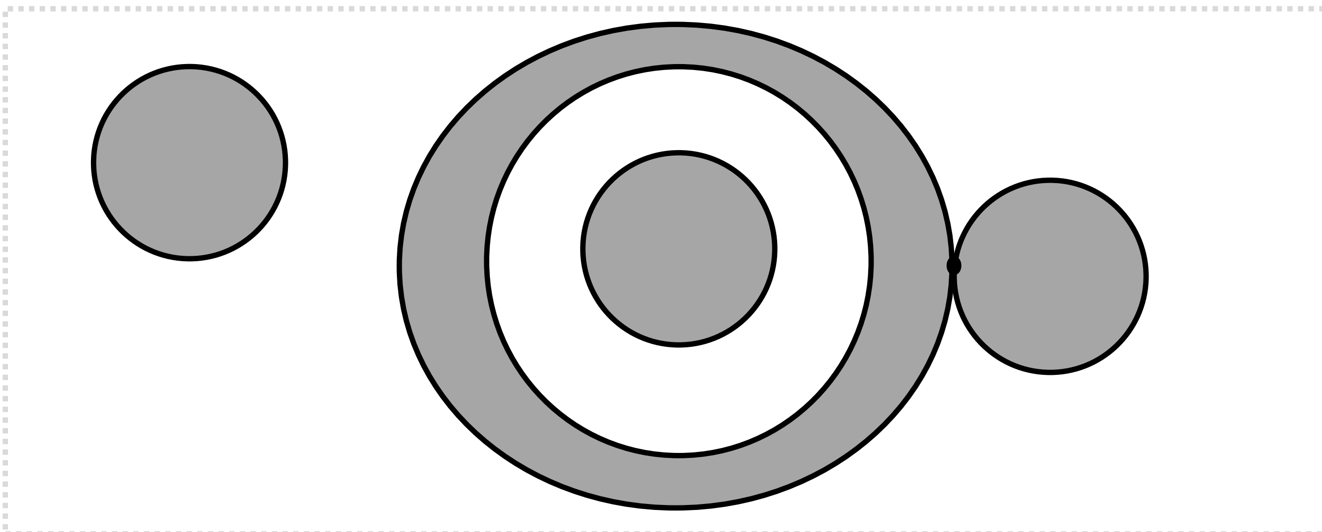
cl(x)

In generalizing the notions of bounds, we want to achieve the effect that the interior of the closure of x is x again. So, the notion of the interior covers the points *between* the bounds, which means that it must be clear for a bound which side of it is in and which is out.

Apartness is the same intuition as before: in the picture below two subspaces are not apart:

x

When we add the bounds we get:

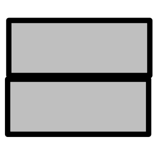


And the interior is not the same as the original:

x'

x had 4 maximal disjoint subspaces, x' has only 3 maximal disjoint subspaces.
Maximal subspaces here can be defined as maximal subsets x where any two points are connected by a continuous function on x (i.e. all the values are in x).

Of course, the same point holds if there are more boundary points: if x consists of two maximally disjoint spaces that are separated by a line of in-between boundary points, the closure adds those points, and the interior is x *plus* the in-between boundary points:

x  cl(x)  int(cl(X)) 

When we go to three-dimensional space $\mathbb{R}^3$ bounds are no longer points or curves but sheets, but the intuition stays the same: regular open sets are sets that are identical to the interior (properly defined) of their closure, and the results that the same as well: the set of regular open sets in $\mathbb{R}^3$ forms a complete atomless Boolean algebra (isomorphic to the one based on $\mathbb{R}$).

This means that the model based on $\mathbb{R}^3$, which I will call the Standard Model, forms a very good model for combining mereological notions (= Boolean notions based on sum operation $\sqcup$) and topological notions, based on three-dimensional continuous space.

At this point, the ideas about how the $\mathbb{R}^3$ model works should be clear enough, so I will not actually go through the effort to define the relevant boundary notions here for $\mathbb{R}^3$.

Apartness and touching

Above gave a model of moments of time, based on the idea that *moment of time* should be, in context, a disjoint set.

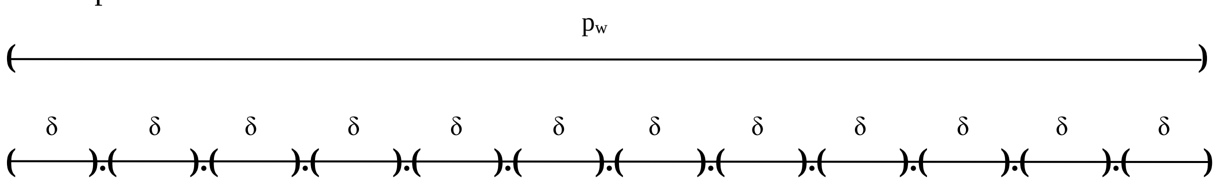
We defined a contextual set of moments $M_{\mathbb{R},w}$

$M_{\mathbb{R},w}$ is a *moment set* iff
1. $M_{\mathbb{R},w}$ is a *partition* of $\mathbb{R}$ into open intervals in $\mathcal{J}$ under $\sqcup$, i.e. MT_w is a disjoint set of open intervals in $\mathcal{J}$ such that $\sqcup M_{\mathbb{R},w} = \mathbb{R}$.
2 *Duration*: for all moments $m_1, m_2 \in M_{\mathbb{R},w}$: **duration**$(m_1)$ = **duration**(m_2) $= \delta_w$, for some $\delta_w > 0$.

So all moments have the same duration δ_w, and intutively, δ_w is the duration below which we cannot make temporal distinctions in context w.

Fact: $M_{\mathbb{R},w}$ omits single isolated points from $\mathbb{R}$
(On the plausible assumption that $M_{\mathbb{R},w}$ contains more than one moment)

If $M_{\mathbb{R},w}$ omited an interval, $\sqcup$ $M_{\mathbb{R},w}$ wouldn't be $\mathbb{R}$.
So the picture is:



In this analysis we use open intervals to model moments of time, because want our contextual moments to be objects in the model, so they need to be regular open sets. What about closed intervals?

Closed intervals do not play a *world building* role in the theory for Boolean reasons. But, of course, a set of moments can have a maximal moment, or be bounded by a moment, so we can define notions of closed or open sets of moments.

This leads to a general moral: we are tempted to think of an orange with peel and an orange without peel in terms of the distinction between a closed object and an open object. But for Boolean reasons we model both via regular open sets. Similarly, we think of two objects touching as occupoying adjacent spaces, and if they are open objects, we naturally think of them sharing a bound.
One reason that this picture is untenable is that we may want to think of two closed objects touching, but that would give them pair of bounds that make a jump (i.e. discreteness), which is not possible in $\mathbb{R}$.
Another reason is the Boolean structure. For linguistic purposes Boolean mereological structures are clearly the better ones, notions of remainder, complement are linguistically relevant and they are Boolean. If we want to have the topological advantages and the Boolean advantages, regular open sets are called for. But that means that notions of touching must be reconstructed in terms of regular open sets: the orange and its peel are both regular open sets and we need a notion of touching that applies to them.

The contextual concerns about moments apply in the same way here. In the macro context we live in, the two slices of cheese touch. But when we zoom in at the atomic or subatomic level, space is created and we *can* unproblematically draw open boundaries between the touching objects.

So, I favor the approach that puts a regular open cushion of space of vanishing size between two objects that touch.

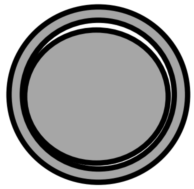
With this, it will be useful to assume that normal objects, *even if they touch* are apart. For that reason it will be useful to define a notion of touching which holds for objects that are apart.

For $x \in S$, cl(x) is the closure of x, **bound**(x) = cl(x) – x.
Let **distance** be the distance function on $\mathbb{R}^3$, (i.e. a function with the obvious metric properties for distance). Let v_w be the 'vanishing value' for distance: a distance value smaller than v_w cannot in w be distinguished from touching.

Let $x_1, x_2 \in S$ and let $x_1 \cup x_2 \in S$ (i.e. x_1 and x_2 are apart).
We assume that if x_1 and x_2 are apart then:
$\forall r_1 \in \mathbf{bound}(x_1)\ \forall r_2 \in \mathbf{bound}(x_2)\colon \mathbf{distance}(r_1, r_2) > 0$

$\text{TOUCH}_w(x_1, x_2)$ iff $\exists r_1 \in \mathbf{bound}(x_1)\ \exists r_2 \in \mathbf{bound}(x_2)\colon \mathbf{distance}(r_1, r_2) < v_c$

In this modelling the fruit may have a peel that touches the fruit. The peel may seems to be like a closed bound. But in the model it isn't. Rather we assume that one object with open bounds (the fruit) is surrounded by another object with open bounds (the peel), and the internal bound of the second touches the external bound of the first:



Of course, we may be more precise and describe a tighter fit between the inner bound of the peel and the outer bound of the fruit: any point in the inner bound of the peel touches some point in the outer bound of the fruit and every point in the outer bound of the fruit touches some point in the inner bound of the peel. That is a very tight fit.

Separation and wholes

Mereotopology, Casati and Varsi 1999, used in a linguistic context by Grimm 2012.
This approach is axiomatic in nature and more tentative, or conservative in nature: obviously, Casati and Varsi want to exclude models in which the central notions they study are blatantly unintuitive (or, if you like, wrong), but they are less willing than I am to jump the gun, and accept the richest axiomatic version as the working theory. They rather think that some conceptual notions that are fixed in the Standard Model may be underdetermined by our philosophical intuitions, and they discuss various variants of mereotopology of different strength in which the relevant notions to be studied can be expressed. A consequence of this, their work is something of a philosophical study into the question: what is the *minimal* mereotopological axiomatic system in which a satisfactory notion of what it means for something to be *one whole object* can be developed.

In the course of this, the definitions involved become rather complex, partly, because in good philosophical tradition the definitions are tailored not to the simple case, but to highly sophisticated cases. They also become complex, because the theory doesn't yet force you into a very limited number of small options concerning these cases.

The following discussion obviously relates to Casati and Varsi 1999, but works out the relevant notions in the Standard Model.

The core question that Casati and Varsi are concerned with is the following. Mereology gives you a theory of entities that are order by a part-of relation and a notion of sum. Since the notion of sum is applicable to any set of entities, there is no difference in mereology between an object that we count as one, a whole, and objects that are merely sums of different objects.

Casati and Varsi propose that we can define in mereotopology the concept of *counting as one whole*. The idea is simple:

▻ An object is *one whole* if it is maximally such that it does not have *separate* parts.

And the mereotopological innovation lies in the definition of *separate* parts.

Now, I am as little inclined to discuss Philosophy here as Casati and Varsi are inclined to discuss semantics, so I take their discussion into the domain of semantics right away.
As far as semantics is concerned, Casati and Varsi can be taken to suggest that the stipulation of Mountain semantics that the denotation of a singular count noun be a set of *atoms*, can be replaced by a conceptual definition.

Let's first give the philosophical notion of *one whole* in the Standard model.

Let $z, z_1, z_2 \in S$, $Z \subseteq S$,

▻ z_1 and z_2 are *disjoint* iff $z_1 \sqcap z_2 \neq 0$
▻ Z is *disjoint* iff for all $z_1, z_2 \in Z$: z_1 and z_2 are disjoint
▻ A *partition* of z is a disjoint set $P_z \subseteq S$ such that $\sqcup P_z = z$.
▻ z *does not have separate parts* iff the only partition P_z such that $\sqcup P_z = \cup P_z$ is the trivial partition $P_z = \{z\}$; otherwise z *has separate parts*.

Your body without your head is not one whole, because, even though it doesn't have separate parts, it is not maximally such, because your body without your head is part of your body with your head and the latter also has no separate parts.

The sum of you and me is not one whole, because it has you as part and me as part, and we are separate.

You are one whole, because there isn't among your parts one that is separate from all your other parts, and if we take the sum of you and anything else, the sum will consist of two separate parts.

In mereotopology we see directly a problem: what about your hat? The hat on your head. It *can* be separated, but *isn't* at the moment. But then, some of your bodyparts also *can* be separated but aren't. This means that effort needs to go into allowing objects to be separate and touch. Because you are one whole and your hat is one whole, but the sum is not one whole.

Casati and Varsi's mereotopology works very hard to define a notion of objects touching , without their sum being *one whole*. In the standard model, this problem is made to disappear, because we maintain that whileyou and your hat indeed touch, you and your hat are are nevertheless apart, and hence separate.

This definition of separtate parts is purely mereological. A topological definition can also be given:

▻ z *does not have separate parts* iff for every $r_1, r_2 \in z$ there is a continuous function inside z from r_1 to r_2.

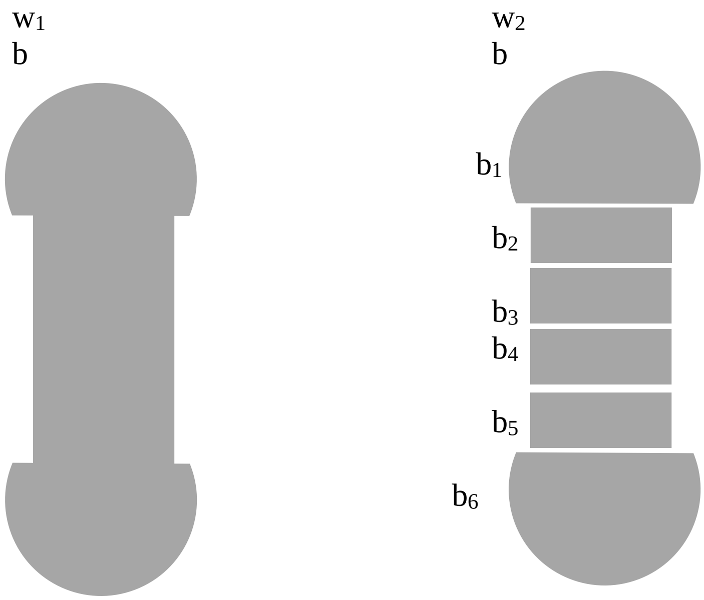
Fact: In the standard model the two definitions are equivalent.

The notion of *whole* then picks out objecs that are maximally this:

▻ z is *whole* iff z does not have separate parts and for every z' such that $z \sqsubset z'$: z' has separate parts.

Example

As an example, look at the two stages of the same object b.
b is a sausage which in w_1 is whole, and in w_2 is cut into slices:



Since b in w_1 and in w_2 consists of the same parts, what makes b whole in w_1 and not whole in w_2?

We assume as a constraint on real slicing that in the sliced sausage the slices are all more than a point apart, even if they touch. This means that $b_{1,\ldots}b_6$ are all more than a point a part, hence separate . And this means that what we derive in w_2 that $\sqcup\{b_1\ldots b_6\} \neq b$, because $\sqcup\{b_1\ldots b_6\} = \cup\{b_1\ldots b_6\}$, which has six separate parts.

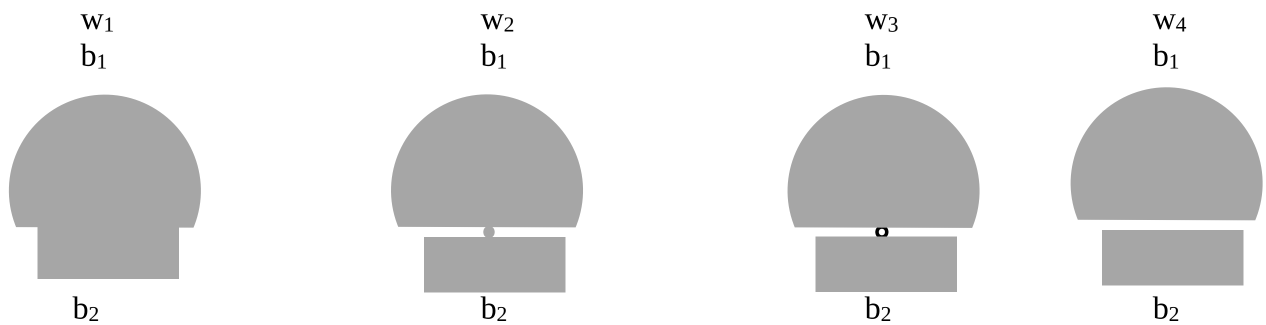
In w_1, the only partition X of b such that $\sqcup X = \cup X$ is {b}, all other partitions by necessity omit bounds: the blocks must be regular open, and together sum up to b, this can only be if the blocks are separated only by bounds, cracks, that will be filled in by $\sqcup$. The point is: the notion of partition needs to satisfy both conditions: the blocks are open and the blocks sum up to b. This is only possible if there are cracks in every non-trivial partition.

This is also true for the part b_1 of b, b_1 also doesn't have separate parts. But b_1 is not maximally so, because it is part of b. So b_1 is not whole, but b is whole in w_1.

We have to accept, then, that the sliced sausage is technically not the same object as the non-sliced object. Before you think this unnatural, realize that this is of course also true for *real slicing*: ever so little stuff disappears in slicing. And what disappears here in one cut is nothing more than a single point deep sheet of points: you cut into an open side and a closed side, remove the relevant bound of the closed side and move them apart, that's all that is needed to turn one whole object into two whole objects.

From the point of view of the standard model it is not a good idea to redefine the notion of touching, so that two separate parts can still touch in one point, because that situation plays a central role in the Boolean structure and you don't want your theory of space to interfere with that.

We can sum the situation up in a different way. In the Standard Model, because all objects are regular (more than a point apart) bounds play a special role in the theory. Look at the following four situations:



Intuitively, we might say the following.
In w_4, b_1 and b_2 are apart, separated, every point in b_1 is more than a single point apart from every point in b_2.
In w_3, b_1 and b_2 are not apart, some point in b_1 is only a point away from some point in b_2. You might say that b_1 and b_2 touch, but don't overlap in this case.
In w_2, b_1 and b_2 are connected by a single point 'land bridge'.
In w_1, finally, b_1 and b_2 form one 'inseparable' whole.
The intuitive description that I give here makes w_1 and w_2 pattern together: in both these cases we have *one connected whole*. And it makes w_3 and w_4 pattern together: in both these cases we have *two wholes*, touching in the one case, not touching in the other.

The point of the Standard Model and the Boolean operation $\sqcup$ is that in the Standard Model the above 'intuitive description' is misleading: objects that are only a single point apart *do not count as two*: they are *a thing with a crack*, which gets removed by $\sqcup$: the operation $\sqcup$ makes situation w_2 and w_3 pattern alike ; $b_1 \sqcup_{w_2} b_2 = b_1 \sqcup_{w_3} b_2$ (with the index on $\sqcup$ indicating the shift in situation).

So, in the Standard Model, because of the Boolean role that apartness plays, we do not want to count situation w_3 as a case where "two" objects touch without overlapping, because then we are forced to regard $b_1 \sqcup b_2$ simultaneously as one inseparably whole (w_2) *and* the sum of two touching non-overlapping objects.

What this means is that there is *tension* between the meriological, Boolean needs of the structure, and the topological needs, which gets resolved by redefining *touching*.

Thus, the Standard Model puts itself even stronger on the side of Sir Patrick Delaney-Podmore than J. K. Rowling does: only situation w_4 is good enough to join the Headless Hunt.

Conceptually count nouns

We move away from philosophy to semantics. As far as semantics goes, there are no wholes, there are no things that count as one whole, or, if there are, they contain non-separated parts that also count as one whole. Take my two kidneys. They are clearly non-separated parts of me, hence neither of my kidneys counts as one whole. Only I do. But do I? Take the ball of space around me with me in it. Can I truly separate this into me and the space around me? Well, I do, by *stipulating* an eigenplace for me, and claim that it is separate from its complement, but with the actual vagueness of the boundary, it is not clear that there is *objective* reality to this. *Vice versa*, a VLBI radio telescope arguably can count as one whole, even though it consists of several separate antennas not connected by cables.

From a semantic point of view, this discussion is rather besides the point, since semantics is not interested in the question of whether my kidneys that are non-separated parts do or don't count as one whole, and whether I do: semantically, the issue of separation and wholeness is always relative to explicit or implicit comparison sets, typically derived from noun denotations. Thus, whether my kidneys are one whole or two wholes is an issue that is only relevant relative to the comparison set, which is the denotation of the noun *kidney*. Thus, the cardinality function that is relevant for count nouns is: $\lambda x.\mathbf{card}_Z(x)$, the partial function that maps elements of S onto natural numbers, relative to a set in terms of which their cardinality is counted.
two kidneys is interpreted as the set of all sums of kindneys that count as 2 relative to the set of kidneys that count as one: $\lambda x.{}^*\text{KIDNEY}_w(x) \wedge \mathbf{card}_{\text{KIDNEY}_w}(x)=2$.

Again, this stipulates one-ness of single kidneys. Mereotopology can do somewhat better than that, though. Let us minimally change the notions of separate parts and wholes to incorporate a comparison set:

▹ An X-*partition* of z is a disjoint set $P_{X,z} \subseteq X$ such that $\sqcup P_{X,z} = z$.

▹ z *does not have separate* X-*parts* iff the only partition $P_{X,z}$ such that $\sqcup P_{X,z} = \cup P_{X,z}$ is the trivial partition $P_{X,z} = \{z\}$; otherwise z *has separate parts*.

▹ z is *one* X iff $z \in X$ and does not have separate parts X-parts and for every $z' \in X$ such that $z \sqsubset z'$: z' has separate parts.

Let us write KIDNEY_w for the union of the denotation of the singular noun *kidney* and the plural noun *kidney*. We are interested in the question whether we can *define* given this set, the set KIDNEY_w which is the denotation of the *singular* noun, using the above definitions.

And the answer is, in many cases, yes.

Take k_1 and k_2 and $k_1 \sqcup k_2$.
k_1 does not have separate KIDNEY_w- parts
$\forall z' \in \mathit{KIDNEY}_w\ [z \sqsubset z' \rightarrow z'$ has separate KIDNEY_w-parts$]$
hence k_1 counts as *one* KIDNEY_w.

Similarly, k_2 counts as *one* KIDNEY_w

But $k_1 \sqcup k_2$ does not count as *one* KIDNEY_w

Given this, we can now define $KIDNEY_w = \lambda x.$ *one* $KIDNEY_w(x)$
and we set: $card_{KIDNEY_w}(x)=1$ iff *one* $KIDNEY_w(x)$
From there on we define the plural denotation as the closure unter sum, $*KIDNEY_w$, which means that we now establish $KIDNEY_w = *KIDNET_w$, and we define $\mathbf{card}_{KIDNEY_w}$ in the usual way: for $x \in *KIDNEY_w$: $card_{KIDNEY_w}(x) = |\{y \in KIDNEY_w: y \sqsubseteq x\}|$

And we get $k_1 \sqcup k_2 \in \lambda x.*KIDNEY_w(x) \wedge \mathbf{card}_{KIDNEY_w}(x)=2$
So far so good.

This works well for many nouns that are conceptually count. That is, actually, not a surprise. In the theories of Rothstein and Landman nouns that are conceptually count. Rothstein focusses on the conceptual atomicity of the denotation of the base denotation of such nouns, their *oneness*, Landman focusses on the conceptual *disjointness* of their base denotation. Disjointness, of course, just means what it says, but *conceptual* disjointness is naturally interpreted as strenghtening disjointness to separation: objects that are conceptually disjoint are separate (*ceteris parisbus*, see Landman 2020 for suggestions on how to deal with exceptional cases where disjointness, and hence separatation, does not hold).

Contextual count nouns

But conceptually count count nouns are only one class of count nouns.
Rothstein 2017 and Landman 2020, based on much earlier work, extensively discuss contextually count count nouns (called *contextually atomic* in Rothstein and *contextually disjoint* in Landman).

Below is Rothstein's meadow. The locals report (1a), while the tax office reports (1b):

(1) a. Of course, the sheep can't get out. The meadow is surrounded by *a fence*.
b. According to our records, four farmers have erected a fence in the area. Each one of these *four fences* falls under the fence-tax law of 1255. One of them is a gated fence, which falls in a separate gated-fence category.

Both these circumstances are perfectly plausible, and they involve different context in which the denotation of the count noun *fence* is made disjoint in different ways. It is similarly not difficult

to set up a context where there are two fences (counting the one with the gate as one and the three parts without as one), or make up a natural story where there are three.
However, claiming that there are, say, 13 fences here (1 of four parts, 4 of one part, 4 of three parts, and 4 of two parts), requires a farfetched context in which the noun denotation exceptionally allows "double counting".

But the fencing structure is the same in situation (1a) and (1b): hence, either the four parts are separate and only touch, and hence count as four whole fences, or they are not separate and they count as one fence. And, in the analysis where count is analyzed through separateness, you can't have it both ways simultaneously.

This is different in the theories in which count is based on disjointness. Here what matters is how you partition the fence. (1a) describes a natural context in which there is a one block partition, and hence one fence, (1b) presents a four block partition, and hence four fences. It is perfectly acceptable on such a theory to assume that in order to make the partition, the context divides the fence parts at the touching sites in a possibly contextually arbitrary way (with respect to which side, say, the connecting bolts go). The partition requires the fencing structure to be dividable into *disjoint* parts that sum up to the whole structure, but not for the structure to be dividable into *separate* parts: the parts need not be separate.

You may think this an issue of little consequence, give that nouns like *fence* can be seen as borderline deviations among count nouns, where the standard is separation. However, such a view becomes untenable once we broaden the discussion **from nouns to noun phrases** and we have to take into account count noun phrases with classifier heads, and in particular **portion readings** of such noun phrases. We are dealing here with a productive area of noun phrase interpretations, and the difference between merely disjoint denotations and separated denotations turns out to be semantically active here. Landman and Khrizman, Landman, Lima, Rothstein and Schvarcz 2015 discusse count *noun phrases* , and in particular the different readings of classifier phrases, like ***glass of wine***. These expression have a variety of readings, several of which are count.

They argue that classifier expressions like *three liters of soup*, *three servings of soup, three plates of soup* have a variety of *semantically distinct* readings, of which some are measure readings that pattern with mass readings, and some are portion readings that pattern with count readings.

three plates of soup

1. A count readding that counts plates:

$$\lambda x.{*}(\lambda z.\,\text{PLATE}_w\,(z) \wedge \text{SOUP}_w(\textbf{contents}_w(z)))(x) \wedge \textbf{card}_{\lambda z.\text{PLATE}_w\,(z) \wedge \text{SOUP}_w(\textbf{contents}_w(z))}(x){=}3$$

The set of sums of three objects each of which is a s plate whose contents is soup.

2. A mass measure reading:

$\lambda x.\text{SOUP}_w(x) \wedge \textbf{plate}_w(x) = 3$

Soup to the amount of three platefuls

[assuming a contextual standard for what volume of soupt counts as one plateful]

3. A count reading that counts separate portions in separate plates:
$\lambda x.{*}(\mathrm{PORTION}_w \cap \mathrm{SOUP}_w)(x) \wedge \mathbf{card}_{\mathrm{PORTION}_w \cap \mathrm{SOUP}_w}(x) = 3 \wedge$
$\forall y \in \mathrm{PORTION}_w \cap \mathrm{SOUP}_w\colon \exists z \in \mathrm{PLATE}_w\colon \mathbf{contents}_w(z)=y$
The set of sums of three portions of soup each of which is the contents of some plate (and hence separated from other soup).

4. A count reading that counts non-separate portions:
$\lambda x.{*}(\mathrm{PORTION}_w \cap \mathrm{SOUP}_w)(x) \wedge \mathbf{card}_{\mathrm{PORTION}_w \cap \mathrm{SOUP}_w}(x) = 3 \wedge$
$\forall y \in \mathrm{PORTION}_w \cap \mathrm{SOUP}_w\colon \mathbf{plate}_w(y) = 1$
The set of sums of three portions of soup each of which measures a plateful of soup.

The difference between readings (3) and (4) is sematically crucial. Casati and Varsi's suggestion can account for reading (3) as a count reading, but it cannot account for reading (4) and a count, rather than a measure reading.

Thus, when I point at the pan and tell you that there are 12 servings/portions in it, this can either be understood as a measure reading or as a non-separate portion reading. The latter can be brought out, because we can set out contextual conditions such that a measure reading is excluded. Thus in our family, we all known that there are two people who are dieting and two who are not, so we all know that there for a meal we need two large servings/portions and two small servings/portions. Now, you come in at the last moment, and brings two more dieting friends and one non-dieting friend, and I say:

(1) Hm, what do I do, I made exactly four servings of soup. How am I going to divide that to serve all of us?

The math problem is not what interests us here, but the original statement:

(1) I made exactly four servings of soup.

The soup is still in the pot, so this is not a separate portion reading. But there is no fixed measure involved, so it is not a measure reading either. The natural assumption is that this is indeed a count portion reading, but one that involved non-separate portions.
On the analyses of Rothstein and Landman, such a reading is to be expected, since the semantic requirement on *portion/serving* is disjointness, which is satisfied (2 small and 2 large disjoint servings make one pot of soup), and *portion* and even *serving* do not *entail* a fxed size.

Hence this is another case of a noun phrase denotation which is disjoint, but not separate, i.e. contextually disjoint.

My conclusion: Casati and Varsi's notion of separateness can play a useful role in semantic analysis, but does *not* enter into the basic semantic mass/count/neat/mess distinctions.